Department of Mathematics
University of British Columbia
MATH 226 Final Exam
December 11, 2012, 12:00PM - 2:30PM

Family Name: ______________________________ Initials: ______

I.D. Number: ________________ Signature: ______________________________

CALCULATORS, NOTES OR BOOKS ARE NOT PERMITTED.

JUSTIFY ALL OF YOUR ANSWERS (except as otherwise specified).

THERE ARE 8 PROBLEMS ON THIS EXAM.

Question	**Mark**	**Out of**
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
Total		80

1.

Let $f(x, y) = x^2 - 2xy + 3y^2$. Let $(x_0, y_0) = (1, 2)$,

(a) At the point (x_0, y_0), find the direction in which f increases most rapidly.

(b) At the point (x_0, y_0), find the set directions in which the rate of increase of f is at least $1/2$ of the maximum rate of increase. Give your answer in terms of the range of angles of deviation from the direction of maximum increase.

(c) Find an equation of the tangent line to the level curve of f passing through (x_0, y_0).

2.

For each of the following subsets of $\mathbb{R}^3$, determine if the set is:

- open
- closed
- bounded

No justification required.

Enter Y for Yes and N for No in the table.

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*
open										
closed										
bounded										

(a) $\{(x,y,z) \in \mathbb{R}^3 : \ 0 < x^2 + y^2 + z^2\}$

(b) $\{(x,y,z) \in \mathbb{R}^3 : \ 3x^2 + 4y^2 + 5z^2 = 1\}$

(c) $\{(x,y,z) \in \mathbb{R}^3 : \ 3x^2 + 4y^2 - 5z^2 = 1\}$

(d) $\{(x,y,z) \in \mathbb{R}^3 : \ 2 < x^2 + y^2 + z^2 \leq 3\}$

(e) $\mathbb{R}^3$

(f) $\{(1,0,0),(0,1,0),(0,0,1),(1,-4,2)\}$

(g) $\{(x,y,z) \in \mathbb{R}^3 : \ y = 0\}$

(h) The x-axis

(i) The complement of the x-axis

(j) The complement of $\{(x,y,z) : x > 0\}$

3.

Compute

$$\int\int_D xy\ dxdy$$

where D is the domain

$$x^2 + y^2 + x \leq \sqrt{x^2 + y^2}, \quad y \geq 0$$

4.

Compute

$$\int\int_D x^2 y \log y + 2x^2 y \; dxdy$$

where D is the domain

$$e^x \leq y \leq e^{2x}, \;\; 1 \leq x^2 y \leq 2.$$

Note: log is the natural logarithm.

5.

Re-write the integral

$$\int_{z=0}^{1}\int_{y=0}^{\sqrt{z}}\int_{x=0}^{\sqrt{z-y^2}} f(x,y,z)\ dxdydz$$

as

(a)

$$\int_{x=\square}^{\square}\int_{z=\square}^{\square}\int_{y=\square}^{\square} f(x,y,z)\ dydzdx$$

(b)

$$\int_{y=\square}^{\square}\int_{x=\square}^{\square}\int_{z=\square}^{\square} f(x,y,z)\ dzdxdy$$

NO justification required.

6.

In this problem, give careful arguments, but you do NOT need to give $\epsilon - \delta$ proofs.

(a) Let

$$f(x,y) = \left\{ \begin{array}{cc} \frac{x\sin(x^2+y^2)+y^3-xy^2}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{array} \right\}$$

i. Compute the partial derivatives of f at (0,0).

ii. At (0,0), is f continuous? differentiable? continuously differentiable ?

(b) For each *real* number $r \geq 0$, consider the function

$$g(x,y) = \left\{ \begin{array}{cc} (x^2+y^2)^r \sin(\frac{1}{x^2+y^2}) & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{array} \right\}$$

Find all values of $r \geq 0$ such that at $(0,0)$, g is

i. continuous

ii. differentiable

iii. continuously differentiable

7.

Prove that the following functions are continuous everywhere by using the $\epsilon - \delta$ definition. Do not use any other properties of limits or continuous functions.

(a) $f(x, y) = \frac{1}{x^2+y^2+1}$

(b) $f(x, y, z) = xyz$

8. Let $f : \mathbb{R}^3 \to \mathbb{R}$ be a differentiable function. Assume that for all (x, y, z), $\nabla f(x, y, z)$ is parallel to the vector $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Show that f is a function of $\rho(x, y, z) = \sqrt{x^2 + y^2 + z^2}$, i.e., there is a function F such that for all (x, y, z), $\quad f(x, y, z) = F(\rho(x, y, z))$.