**Math 223, Additional problems: Sets and maps. Do not hand in.**

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**Basic problems: these should be easy!**

1. Let $A_n = \left[0, \frac{1}{n}\right] \times [0, n]$. Draw the picture representing $\bigcup_{n\in\mathbb{N}} A_n$ and $\bigcap_{n\in\mathbb{N}} A_n$. (Both should be subsets of $\mathbb{R}^2$).

2. Let $T_a = \{x \in \mathbb{R} : x \geq 0 \text{ and } x < a - 2\}$. Prove that if $T_a = \varnothing$ then $a \leq 2$.

   What is wrong with the following start of an argument:

   > "Since $x \geq 0$ and $x < a - 2$ we must have $0 \leq x < a - 2$. Then ... "

3. Prove that $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$ (one of DeMorgan laws for sets), and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

4. Prove or disprove: $(A \cup B) \setminus B = A$.

5. Write the converse, contrapositive, and the negation of the following statement:

   > If the collection of vectors $v_1, \dots v_n$ is *spanning*, then for every $v \in V$ there exist real numbers $a_1, \dots, a_n$ such that $v = a_1 v_1 + \cdots + a_n v_n$.

   *Remark:* It does not matter what 'spanning' means, and what the expression $v = a_1 v_1 + \cdots + a_n v_n$ means; this is simply an exercise in formal language, you should be able to write these statements without knowing the definitions of all these terms.

6. Fill in the details in the proof on p. 3 of the notes for January 11.

7. Prove that if $f : A \to B$ is injective and $g : B \to C$ is injective, then $g \circ f : A \to C$ is injective. Is the converse statement true?

8. Does the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \tan(x)$ have an inverse? How can we modify this function to make the inverse function well-defined?

9. True or false: $\arcsin(\sin(x)) = x$ for all $x \in \mathbb{R}$? What about $\sin(\arcsin(x)) = x$ for all $x \in [-1, 1]$? Write both statements in terms of composition of functions.

10. Explain why multiplication by 2 defines a bijection from $\mathbb{R}$ to $\mathbb{R}$, but not from $\mathbb{Z}$ to $\mathbb{Z}$. *Remark: Even though $f$ and $g$ are defined by the same formula (multiplication by 2), they are different functions because their domains and codomains are different.*