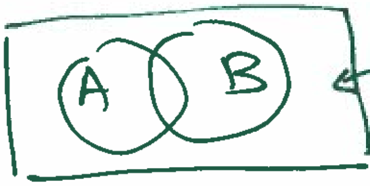
Office Hours
11:30 - 1 W
12:30 - 2 F

MATH 217

Today: sets, functions, maps.

Last time: sets.
defined unions, intersections.

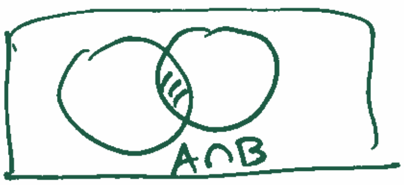
More on unions and intersections

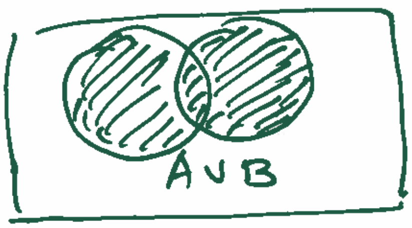


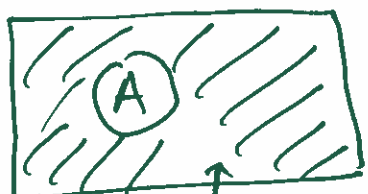
← universe (universal set).

← meant in order to avoid "set of all sets"

(Exer: think about: if you had the set of all sets, would it be an element of itself?)



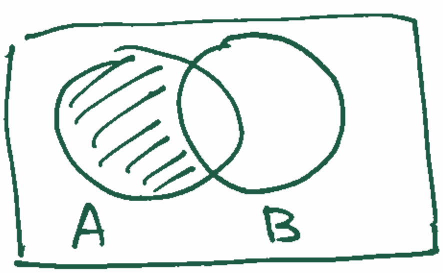




↑ the complement of A

(denoted by $\overline{A}$)

(needs the universal set to be precise)



Def: difference of sets: $A \setminus B$

↑ \setminus

$$A \setminus B = \{x : x \in A, \ x \notin B\}.$$
$$\shortparallel$$
$$A \cap \overline{B}.$$

Notation: if you have several sets $A_1, \dots, A_n$ their union is denoted by

$$\bigcup_{k=1}^{n} A_k = A_1 \cup A_2 \cup \dots \cup A_n = \{x : \exists k \in \{1, \dots, n\}, \ x \in A_k\}.$$

exists

Similarly, $\bigcap_{k=1}^{n} A_k = A_1 \cap \cdots \cap A_n$

$= \{x : x \in A_1, x \in A_2, \ldots, x \in A_n\}$

$= \boxed{\{x : \forall\, k \in \{1, \ldots, n\} \;\; x \in A_k\}}.$

or $\forall\, k : 1 \le k \le n$

Similarly, can define the union / intersection of <u>any</u> indexed collections of sets:

if the set of indices is <u>countable</u>, we write

$$\bigcup_{n=1}^{\infty} A_n = \{x : \exists\, n \in \mathbb{N},\ x \in A_n\}$$

<u>Generally</u> let I be a set of indices

let A_x be a collection of sets,

one for each $x \in I$

Then $\bigcup_{x \in I} A_x = \{y : \exists\, x \in I,\ y \in A_x\}.$

$\bigcap_{x \in I} A_x = \{y : \forall\, x \in I,\ y \in A_x\}.$

<u>Example</u> Let $A_x = \{y : 1 \le y \le x\}$ for $x \in \mathbb{R}$.

Find $\bigcup_{x \in \mathbb{R}} A_x$.

$\boxed{\text{Answer: } [1, \infty)}$

What does A_x look like?

if $x < 1$, then $A_x = \emptyset$

if $x \ge 1$

$A_x = [1, x]$

(if $x = 1$, $A_x = \{1\}$)

Formally, to prove that $\bigcup_{x \in \mathbb{R}} A_x = [1, \infty)$

we need to prove two things:

every $\cancel{\forall}$ $y \in \bigcup_{x \in \mathbb{R}} A_x$ also belongs to $[1, \infty)$

and conversely, every $y \in [1, \infty)$ also belongs to $\bigcup_{x \in \mathbb{R}} A_x$.

exer.

A note about "conversely":

the assumption $\;P \Rightarrow Q\;$ the conclusion "P implies Q"

"if P then Q"

$\underbrace{\text{statements}}_{\text{an implication}}$

a new statement which is true in the following cases:

P	Q	P => Q
T	T	T
T	F	F
F	T	T
F	F	T

The converse is $Q \Rightarrow P$.

Do not confuse with: contrapositive

"not Q => not P" $\qquad$ is equivalent to

$\sim Q \Rightarrow \sim P$ $\qquad$ the original statement $P \Rightarrow Q$.

Cartesian product of sets

Def: A, B - sets.

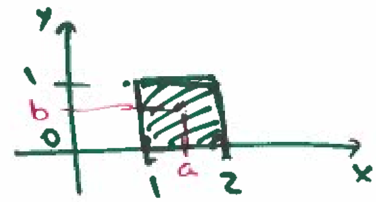
$$A \times B = \{ (a,b) \mid a \in A,\ b \in B \}.$$

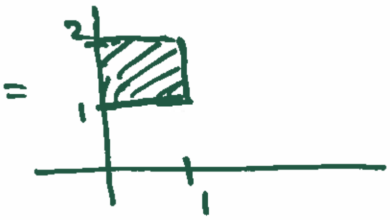
\times

ordered pairs of elements.

Example $A = [1,2]$, $B = [0,1]$

interval in $\mathbb{R}$

$A \times B =$ 

$B \times A =$ 

pictures of $A \times B$ as a subset of $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$

$= \{ (x,y) \mid x \in \mathbb{R},\ y \in \mathbb{R} \}$

$\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$ - familiar "3-space"

Def: $A_1, \dots, A_n$ - a finite collection of sets.

$$A_1 \times \dots \times A_n \overset{\text{def}}{=} \{ \underbrace{(a_1, \dots, a_n)}_{n\text{-tuple}} \mid a_i \in A_i \}.$$

Functions (Maps)

$f : A \to B$ - a function is an assignment of <u>one</u> element of B (called $f(x)$) to every element x of A.

domain of the function f.

codomain