Linear Algebra

Text:
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HW: Mon 10 am on Canvas.
typeset.

- Linear transformations of vector spaces.
  will focus on language, proofs.
  will have computation projects.

First Maps ⟷ and Sets.

Set: a collection of elements.

How to specify a set:

1. List all its elements:

set notation.

Ex: ① $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$

$\mathbb{N}$: name of the set - the set of natural numbers.

$\{ \ldots \}$: elements of the set.

② Let $E = \{2, 4, 6, 8, 10, 12, \ldots\}$ - set of even natural numbers

2. Write down the condition defining the set:

Ex: $E = \{ n \in \mathbb{N} : n \text{ is even} \}$

$\in$: is an element of

Notation: $\{ \text{name an element} \mid \text{condition on } n \}$

name an element (n)

:

in our example, $26 \in E$, but $27 \notin E$.

\in

\notin - TeX command

Standard names for certain sets

$\emptyset$ - emty set. - a set with no elements.

Example How to build everything from $\emptyset$ and set notation:

$\emptyset$, make $\{\emptyset\}$ - the set of one element: the empty set.

$\{\emptyset, \{\emptyset\}\}$ - set of two elements: $\emptyset$, and the set whose only element is the empty set.

Why $\{\emptyset, \emptyset\}$ = $\{\emptyset\}$ doesn't work: in the set notation, doesn't matter how many times you list an element, it counts once.

this way we can define the integers.

Names for the common sets

$\mathbb{N}$ - the natural numbers = positive integers.

$\mathbb{R}$ - reals

$\mathbb{Q}$ - rational numbers

$\mathbb{Z}$ - integers

Subsets: $A \subset B$ - "A is a subset of B" if every element of A is an element of B.

$\forall x \in A, x \in B$

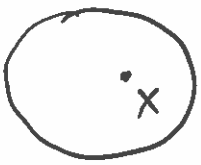
$\forall$ - the universal quantifier "for all"

we have:

$\emptyset \subset A$ for any set A

$\emptyset \subset \mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$.

Venn diagrams



← set A, x - an element of A

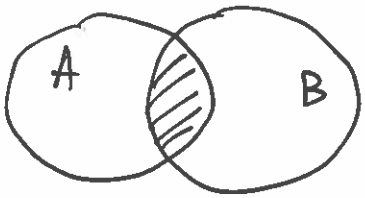
- "A is a subset of B".

Def: $A \cap B$ - the intersection of A and B

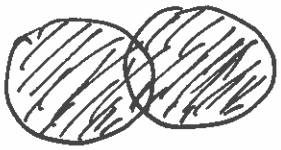
$= \{ x \mid x \in A \text{ and } x \in B \}$.

$A \cup B = \{ x \mid x \in A$ or $x \in B \}$

↑ includes when $x \in A \cap B$.



$A \cap B$



$A \cup B$