Last time:
Matrices

Composition of lin. maps $\leftrightsquigarrow$ product of matrices.

lin. map $\longrightarrow$ matrix
$A: V \to W$ $\quad\uparrow$ choice of basis of V and W

A little more about linear maps

Example V - n-dim. vect. space over F; think $V = F^n$.

$Id: V \to V$ standard basis $\{e_i\}$ of V
doesn't matter which basis, in this case

Matrix of the Identity map:

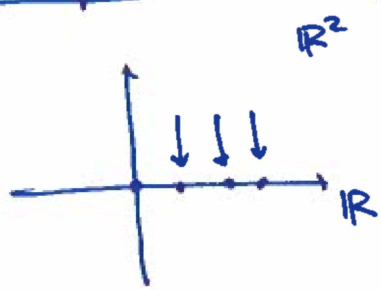
$$\begin{pmatrix} 1 & 0 & & 0 \\ 0 & 1 & & 0 \\ 0 & 0 & & 0 \\ 0 & & & \\ 0 & 0 & & 1 \end{pmatrix} n$$
n

"diagonal"

— $\boxed{\text{diagonal matrix with 1's on the diagonal}}$

"identity matrix".

Example (!!) Projectors.



$P: \mathbb{R}^2 \to \mathbb{R}$
$(x, y) \longmapsto x$

maps the plane to the x-axis.
the whole y-axis maps to $\bar{0} \in \mathbb{R}$.

Def: If $A: V \to W$ is a linear transformation,
then $\mathrm{Ker}(A) = \{v \in V \mid A \cdot v = \bar{0}_{\text{in } W}\} < V$

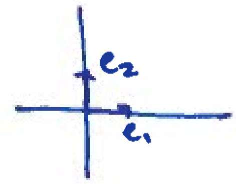
kernel of A $= A^{-1}(\bar{0})$
in the sense of functions — "pre-image of $\bar{0}$".

$\mathrm{Im}(A) = \{A \cdot v \mid v \in V\} < W$

Image of A (same as the image or range in the sense of functions)

In our example $P : \mathbb{R}^2 \to \mathbb{R}$,

$\ker(P) = $ the y-axis $= L(e_2)$

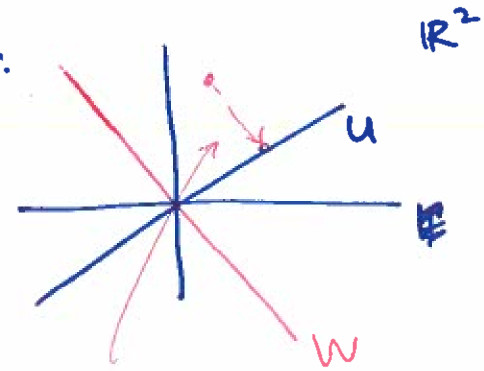


$\operatorname{Im}(P) = \mathbb{R}$.

Fact: (exer.) For a linear transf., $\ker(A)$ and $\operatorname{Im}(A)$ are linear subspaces of V and W, respectively.

Def: $\dim(\operatorname{Im}(A))$ is called the **rank** of A.

Example:



go parallel to W until you hit U.

Projector onto U along W: lin. op. from $\mathbb{R}^2$ to $\mathbb{R}^2$

Its image is U

Its kernel is W.

Read 4.2.