

Homework 6: Determinants, Part 1.
Due Thursday March 2.

1. Compute the following determinants:

(a)

$$\begin{vmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ -1 & 1 & 0 \end{vmatrix}$$

(b)

$$\begin{vmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 1 \end{vmatrix}.$$

2. Problem 6.2 from Jänisch

3. Find the $n \times n$ -determinant

$$\begin{vmatrix} \lambda & 1 & 0 & \dots & 0 \\ 0 & \lambda & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & 0 \\ 0 & \dots & 0 & \lambda & 1 \\ 0 & \dots & 0 & 0 & \lambda \end{vmatrix}$$

where λ is a scalar (it should be a function of λ).

4. Let a_0, \dots, a_{n-1} be scalars. Prove that

$$\begin{vmatrix} -t & 0 & 0 & \dots & 0 & -a_0 \\ 1 & -t & 0 & \dots & 0 & -a_1 \\ 0 & 1 & -t & \dots & 0 & -a_2 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & 1 & -t & -a_{n-2} \\ 0 & \dots & \dots & \dots & 1 & -(t + a_{n-1}) \end{vmatrix}$$

equals $(-1)^n(t^n + a_{n-1}t^{n-1} + \dots + a_1t + a_0)$ (as a function of t).

Hint: Use the expansion with respect to the last column.