Math 223, Homework 1: Sets and maps. Due January 16.

1. Let A, B, C be sets. Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

2. Let $f : A \to B$ be a function.

   (a) Prove that for A_1, A_2 subsets of A, $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$. Is the same statement true with $\cup$ replaced with $\cap$?

   (b) Prove that $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$ for B_1, B_2 -subsets of B. Is the same statement true with $\cup$ replaced with $\cap$?

3. Problem 1.1 from Jänisch

4. Problem 1.2 from Jänisch

5. Problem 1.3 from Jänisch

6. Let A and B be finite sets, and denote by B^A the set of all functions from A to B. Prove that $\#(B^A) = (\#B)^{(\#A)}$, where $\#A$ denotes the number of elements of A.

Practice problems, not for handing in.

7. Do the "Test" for Chapter 1 (Section 1.3) in Jänisch.

8. Let $\mathcal{P}(A)$ be the set of all subsets of a set A (it is called the *power set* of A).

   (a) Let $A = \{\varnothing, 1, \{1\}\}$. List all the elements of $\mathcal{P}(A)$.

   (b) For a subset B of A, define the *indicator function* of B by

   $$\chi_B(x) := \begin{cases} 1, & \text{if } x \in B \\ 0, & \text{if } x \notin B. \end{cases}$$

   Let $C = \{0, 1\}^A$ be the set of all functions from A to the set $\{0, 1\}$. Define a bijective function from $\mathcal{P}(A)$ to C.

   (c) Prove that for any set A, $\#\mathcal{P}(A) = 2^{\#A}$.