
(solutions will be posted on Wednesday)

1. Determine the range of the functions $f : \mathbb{R} \to \mathbb{R}$ defined as follows:

   (a) $f(x) = \frac{x^2}{1 + x^2}$

   (b) $f(x) = \frac{x}{1 + |x|}$

2. Let $f : \mathbb{N} \times \mathbb{N} \to \mathbb{R}$ be defined by

   $$f(a, b) = \frac{(a + 1)(a + 2b)}{2}$$

   Show that the image of $f$ is contained in $\mathbb{N}$, so that $f : \mathbb{R} \to \mathbb{N}$ is a well-defined function.

3. Explain why multiplication by 2 defines a bijection from $\mathbb{R}$ to $\mathbb{R}$, but not from $\mathbb{Z}$ to $\mathbb{Z}$.

4. Write four different bijections $f : \mathbb{N} \to \mathbb{N}$.

5. Final Exam - Dec 2010 Prove that the following function is bijective

   $$f : \mathbb{R} - \{-2\} \to \mathbb{R} - \{1\}$$

   defined by $f(x) = \frac{x + 1}{x + 2}$