Lecture 1
Jan 7
2020

- Math 220

real math:
reasoning, communication

- it's not about "math" - formulas
numbers

but language

- ways to express statements rigorously
prove things are true...

Goals: to start speaking the same language.
we will examine mathematical statements
and their proofs, proof techniques, ...
and foundations for the math we already know.

- examine building materials, rules for mathematics.

Rules: 1) Never write or say anything that doesn't make sense to you
2) Be as precise as you can.

Today: - Sets (1.1)
- Logic (2.1, 2.2)

Assignment: Read, highlight things you do not understand.

Goal: start being able to read matematics.

Sets: "set" - the only word in math that doesn't have a formal definition.

set - a collection of objects

can be numbers
or other sets
or people, or...

Notation: $\{1, 2, 3, 4, 5\}$ - set of numbers:
its elements are 1, 2, 3, 4, 5.
(5 elements).

indicates a set.

Examples: 0) $\{0, \pi, e, 4\}$ - set of 4 elements.
(also a set of numbers)

could also be a strange set with two letters and two numbers as elements.

1) $\mathbb{N}$ - natural numbers ← our convention: $0 \notin \mathbb{N}$

$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$ - infinite.

$\mathbb{Z} = \{\ldots -2, -1, 0, 1, 2, 3, \ldots\}$ - all integers

2) $\mathbb{R}$ - real numbers - how to define them?
cannot list the elements ← will prove

To define $\mathbb{R}$ takes a course...
Think of it as points on the line (for now).

Notation: A - set

"belongs to", "is an element of"

~~$a \in A$~~ $a \in A$ ← a is an element of the set A

Examples: $1 \in \mathbb{N}$

an element, a set

$0 \notin \mathbb{N}$ - 0 is <u>not</u> a natural number.

Describing sets

Way 1) List all its elements.

Works well for finite sets, some infinite sets (using ....)

$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$ - ok

$\mathbb{R} = \{$ ~~1.1, 0.25, 3.1415,~~ $\ldots \}$ - not OK.

Way 2: Describe the rule for membership in the set:

set-builder notation: $\{\ \ :\ \ \}$

elements, rule

Let $E = \{ n \in \mathbb{N} : n \text{ is even} \} = \{2, 4, 6, 8, \ldots\}$

the set of all even natural numbers.

such that or "satisfying the condition"

Examples of mathematical statements (suggested by the students)

Ex 1: $\frac{\pi^2}{6} = \underbrace{1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \ldots}_{\text{what does this mean?}} = \sum_{n=1}^{\infty} \frac{1}{n^2}$

this equality is true

hard theorem (Euler?)

Def'n is part of calculus

notation (this is the definition of the RHS)

Ex 2: $e^{\pi i} = -1$ — Euler identity. (here $i := \sqrt{-1}$)

(equivalent to $\begin{cases} \cos(\pi) = -1 \\ \sin(\pi) = 0 \end{cases}$)

real issue: what does i mean?
what does $e^{\pi i}$ mean??
(how to exponentiate numbers?)

Statements:

Logic - study of truth of statements.

Statement is a sentence that has truth value: true or false.

Examples: • This product may contain harmful materials
— is NOT a statement. (mathematically)
— cannot tell true or false!

(exer: take any news article, count non-statements).

Start with simple statements, make more complicated ones:

example: Let P be the statement
"$\cos(\pi) = -1$"
let Q be the statement "$\sin(\pi) = 0$"

Both are true.

Can form the statement "P and Q":

"$\cos(\pi) = -1$ and $\sin(\pi) = 0$".

Ex: $a^2+b^2=c^2$ – as it is, a non-statement. (we did not define a,b,c)

"Let a,b be the lengths of the sides of a right triangle, c = length of the hypotenuse. Then $a^2+b^2=c^2$" — statement (true).

Jumping ahead: predicate – a statement that depends on variables: when you plug in values for the variables, it becomes true / false.

$P(a,b,c)$: "$a^2+b^2=c^2$" is a predicate (a,b,c are variables):

e.g. $P(3,4,5)$ is true
$P(1,2,3)$ is false.

"Calculus of statements"

Logical operations: AND, OR, NEGATION

AND ↑ "conjunction"
OR ↑ "disjunction"

P, Q – statements.

Def: $P \wedge Q$ = P and Q : statement, whose truth is defined as follows:

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

← truth table

This defines a logical operation!

Def: $P \vee Q$ - "P or Q" - disjunction

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

← (not exclusive)

Def: $\neg P$ - "not P" negation:

P	$\neg P$
T	F
F	T

Assignment: Practice negating statements:

"Every student knows how to negate any statement" ← negate that :)