1. (a) Prove that there are infinitely many primes $p$ such that $p \equiv 3 \mod 4$. *Hint: try to proceed the same way as in Euclid’s proof of the statement that there are infinitely many prime numbers; but instead of making the number $N = p_1, \ldots, p_n + 1$, make a number $N$ that is definitely congruent to 3 modulo 4 (and that still differs by 1 from a number that is divisible by all of $p_1, \ldots, p_k$).*

(b) Could this proof have worked for the primes congruent to 1 modulo 4?

2. Prove that the number 123456782 cannot be represented as $a^2 + 3b^2$ for any integers $a$ and $b$. *(Hint: Consider the remainder mod 3).*

3. Prove that there do not exist integers $a$, $b$ and $c$ such that

$$12345678910111213 = a^2 + 25b^2 + 5c^2.$$