A detailed list of topics to review for Math 220 Midterm 2, on March 11.

Basics: Negating complex sentences, negating implications, forming converses and contrapositives, DeMorgan laws.

Chapter 4:

(1) Basic proofs involving sets (Sections 4.4). You need to know the definitions: for example, what does it mean that $A \subseteq B$? Or how to prove that two sets A and B are equal? (you need to prove both inclusions $A \subseteq B$ and $B \subseteq A$).
(2) Basic properties of set operations (Section 4.5).
(3) Cartesian products of sets (section 4.6).
(4) Congruences of integers (Section 4.2, but see also Workshop 3 problems and homework). You need to know the definitions of divisibility (what does it mean that $a|b$?), congruence $\mod m$, and the basic properties of congruences (how do congruences behave with respect to addition/multiplication). You need to be able to make simple arguments based on case-by-case analysis and properties of congruences; in particular, make sure you know how to prove statements such as "any perfect square is congruent to 0 or 1 $\mod 4$").

Chapter 5:

(1) Proof that a statement is false by means of a counterexample. (section 5.1).
(2) Proof by contradiction (Sections 5.2-5.3). In particular, proof by contradiction can include proofs involving rational/irrational numbers.
(3) You need to know how to prove that there are infinitely many prime numbers.

Chapter 6:

(1) The notion of a well-ordered set (you need to know the definition, and be able to come up with examples of sets that are well-ordered, and sets that are not well-ordered). (section 6.1)
(2) Well-ordering of $\mathbb{N}$, and the principle of mathematical induction. You need to know and understand the statements of the well-ordering axiom, and of the principle of mathematical induction and its variations (e.g. strong induction).
(3) You need to be able to prove various statements using induction or strong induction. The types of statements include: formulas for sums; inequalities; statements about divisibility or congruence; statements about sets (such as the cardinality of the set of

all subsets of a given set); recursively defined sequences. This is the material in Sections 6.2 and and 6.4, but please also look at your lecture notes and the workshops.

Graph theory:

(1) The definition of a simple graph
(2) The fact that the number of edges in a simple graph is half the sum of degrees of ties vertices
(3) Connectedness of graphs, the notion of a connected component of a graph.
(4) The notion of an Euler walk in a graph.