A detailed list of topics to review for Math 220 Midterm 1, on February 5.

Chapter 1:

(1) The notions of a set, subset, element. Please make sure that you clearly understand the distinction between the usage of the symbols '$\in$' and '$\subseteq$', and also do not confuse things such as $\emptyset$ and $\{\emptyset\}$. In particular, please check that you understand why writing $\{1\} \subseteq \{1, 2, 3\}$ is correct, and $1 \in \{1, 2, 3\}$ is correct, but $1 \subseteq \{1, 2, 3\}$ is incorrect.
(2) The notion of a power set.
(3) Set operations: union, intersection, difference, complement. Venn diagrams.
(4) Direct product of sets.

Chapter 2:

(1) Statements.
(2) Logical operations – conjunction, disjunction, negation.
(3) Open sentences. You should understand the relationship between open sentences and sets: let $P(x)$ be an open sentence, where the domain for the variable x is a set U. Then we think of U as the universal set. Then, we can associate a subset of U with the statement P – it is the set A_P defined as $A_P := \{x \in U \mid P(x) \text{is true}\}$. Thus, every open sentence *partitions* the domain into two disjoint sets: the set A_P of all elements x such that $P(x)$ is true, and the set of all x such that $P(x)$ is false. With this correspondence between sets and sentences, conjunction of statements corresponds to the intersection of sets, disjunction corresponds to union of sets, and negation corresponds to taking the complement. You should understand this statement.
(4) Logical equivalences and truth tables. You should be able to construct truth tables for compound statements, and use them to check if two compound statements are logically equivalent.
(5) Some important logical equivalences, for example, DeMorgan laws (both for statements and sets). This means, you should be able to correctly negate complex statements involving conjunctions, disjunctions and negations. Distributivity laws.
(6) The implication. Converse and contrapositive. The fact that $P \Rightarrow Q$ is logically equivalent to $\sim P \vee Q$. Logical equivalence of the implication and the contrapositive. Please note: there are 4 different things: the implication $P \Rightarrow Q$; the converse $Q \Rightarrow P$; the contrapositive $\sim Q \Rightarrow \sim P$; and the negation of the

implication itself, $\sim (P \Rightarrow Q)$. Of these, only the implication and its contrapositive are logically equivalent. Otherwise, none of these implies the other. In particular, if $P \Rightarrow Q$ is true, it does not at all mean that the converse should be true (see if you can come up with examples). The negation of the implication $\sim (P \Rightarrow Q)$ is logically equivalent to $\sim (\sim P \vee Q) \equiv P \wedge \sim Q$. In particular, please do not confuse this with the contrapositive or converse, or the contrapositive to the converse (which is a very popular mistake – write down the contrapositive to the converse, and you will see why). You should be able to come up with verbal examples, such as "if I do not buy a lottery ticket, I will not win the lottery". Try making the converse, contrapositive, and the negation of this statement, and look at the differences.

(7) The biconditional.
(8) Quantifiers. Unions and intersections (and complements) of indexed unions of sets. Negation of statements involving quantifiers.

Chapter 3, and 4.4. We will deepen the study of these proof techniques later, but you should be able to prove statements about sets, and also simple statements related to the integers, using the following techniques:

(1) Direct proof
(2) Disproving statements by a counterexample.

The statements you will be expected to prove (or disprove) will be either about relationships between different sets, or about unions or intersections of indexed collections of sets, or about even/odd integers.