1. **Question 2.68**. State the negation of the following quantified statements:
   (a) For every rational number r, the number $1/r$ is rational.
   (b) There exists a rational number r such that $r^2 = 2$.

2. Let P be the statement '$\forall x \in \mathbb{R}\, \exists y \in \mathbb{R}, y^2 = x$'.
   (a) State this in words (no notation allowed).
   (b) Is this statement true?
   (c) State the negation in symbols.

3. **Modified Question 2.70** Determine the truth value for each of the following statements.
   (a) $\exists x \in \mathbb{R}, x^2 - x = 0$.
   (b) $\forall x \in \mathbb{R}, \sqrt{x^2} = x$.
   (c) $\exists x \in \mathbb{R}, \exists y \in \mathbb{R}, x + y + 3 = 8$.
   (d) $\forall x, y \in \mathbb{R}, x + y + 3 = 8$.
   (e) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x + y + 3 = 8$.
   (f) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, x + y + 3 = 8$.
   (g) $\exists m, n \in \mathbb{N}, n^2 + m^2 = 25$.
   (h) $\forall m \in \mathbb{N}\ \exists n \in \mathbb{N}, n^2 + m^2 = 25$.

4. **Question 2.72.** Let $P(x)$ and $Q(x)$ be open sentences where the domain of the variable x is a set S. Which of the following implies that $\sim P(x) \Rightarrow Q(x)$ is false for some $x \in S$?
   (a) $P(x) \wedge Q(x)$ is false for all $x \in S$.
   (b) $P(x)$ is true for all $x \in S$.
   (c) $Q(x)$ is true for all $x \in S$.
   (d) $P(x) \vee Q(x)$ is false for all $x \in S$.

5. Let $S = [1, 2]$ and $T = (3, \infty)$. Determine the truth value of the following statements, and state each of them in words. Prove your answers— **no proof = no marks**
   (a) $\exists x \in S$ s.t. $\exists y \in T$ s.t. $|x - y| > 3$
   (b) $\exists x \in S$ s.t. $\forall y \in T, |x - y| > 3$
   (c) $\forall x \in S, \exists y \in T$ s.t. $|x - y| > 3$
   (d) $\forall x \in S, \forall y \in T, |x - y| > 3$

6. Let $I = \{n^2 \mid n \in \mathbb{Z}\}$. Let P be the statement

$$\bigcup_{k \in I} [k, 2k] = \mathbb{R}.$$

   (a) Reformulate P using quantifiers.

(b) Is P true or false? Prove or disprove.

7. Let $A = \{x \mid \forall n \geq 3,\ 1/n < x < 1 - 1/n\}$.

   (a) Represent the set A as an intersection of an indexed collection of sets.

   (b) Find A (that is, represent it in the simplest possible form, using the notation for intervals in $\mathbb{R}$). No proof required.

8. Suppose you have the following information about the population of the planet QE220:

   - Among the inhabitants of QE220 who can watch TV, not all have antennae on their head.
   - The inhabitants of QE220 that are green and do not have antennae, cannot watch TV.

   Does it follow that not all the inhabitants of QE220 that can watch TV are green? Justify your answer.
   Hint. Start by writing down the statement you are asked about.

9. **Question 3.8.** Prove that if x is an odd integer, then $9x + 5$ is even.