

1. (**Modified Question 2.2, p. 65**) Consider the sets  $A$ ,  $B$ ,  $C$ , and  $D$  below. First, describe the sets  $A$  and  $D$  using a complete sentence ("  $A$  is the set of numbers which..."). Then determine which of the following statements are true: Give an explanation for each *false* statement.

$$A = \{1, 4, 7, 10, 13, 16, \dots\} \quad B = \{x \in \mathbb{Z} : x \text{ is odd}\}$$

$$C = \{x \in \mathbb{Z} : x \text{ is prime and } x \neq 2\} \quad D = \{1, 2, 3, 5, 8, 13, 21, 34, 55, \dots\}.$$

- (a)  $25 \in A$   
 (b)  $22 \in A \cup D$   
 (c)  $C \subseteq B$   
 (d)  $\emptyset \in B \cup D$
2. (**Modified Question 2.6, p. 65**) For the open sentence  $P(A) : A \subseteq \{1, 2, 3\}$  over the domain  $S = \mathcal{P}(\{1, 2, 4\})$ , determine:
- (a) all  $A \in S$  for which  $P(A)$  is true  
 (b) all  $A \in S$  for which  $P(A)$  is false  
 (c) Let  $W$  be the set of all  $A \in S$  for which  $A \cap \{1, 2, 3\} = \emptyset$ . List all the elements of  $W$ . Then find the intersection of  $W$  with the set of all  $A \in S$  for which  $P(A)$  is true.
3. (**Question 2.14, p. 66**) State the negation of each of the following statements:
- (a) At least two of my library books are overdue.  
 (b) One of my two friends misplaced his homework assignment.  
 (c) No one expected that to happen.  
 (d) It's not often that my instructor teaches that course.  
 (e) It's surprising that two students received the same exam score.
4. (**Question 2.16, p. 66**) For the sets  $A = \{1, 2, \dots, 10\}$  and  $B = \{2, 4, 6, 9, 12, 25\}$ , consider the statements

$$P : A \subseteq B. \quad \text{and} \quad Q : |A - B| = 6.$$

Which of the following statements are true:

- (a)  $P \vee Q$   
 (b)  $P \vee \sim Q$   
 (c)  $P \wedge Q$   
 (d)  $(\sim P) \wedge Q$   
 (e)  $(\sim P) \vee (\sim Q)$ .
5. Prove the distributive law  $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$  by constructing the truth table. (Here  $P$ ,  $Q$ , and  $R$  are statements).

6. (**Question 2.20, p. 67**) For statements  $P$  and  $Q$ , construct a truth table for  $(P \Rightarrow Q) \Rightarrow (\sim P)$ .
7. (**Question 2.24, p. 67**) Two sets  $A$  and  $B$  are non-empty disjoint subsets of a set  $S$ . If  $x \in S$ , which of the following are true? (If false, give an example when the statement does not hold, called "counterexample").
- (a) It is possible that  $x \in A \cap B$ .
  - (b) If  $x$  is an element of  $A$ , then  $x$  can't be an element of  $B$ .
  - (c) If  $x$  is not an element of  $A$ , then  $x$  must be an element of  $B$ .
  - (d) It's possible that  $x \notin A$  and  $x \notin B$ .
  - (e) For each nonempty set  $C$ , either  $x \in A \cap C$  or  $x \in B \cap C$ .
  - (f) There exists a nonempty set  $C$ , such that both  $x \in A \cup C$  and  $x \in B \cup C$ .
8. (**Question 2.28, p. 68**) Consider the statement (implication):  
*If Bill takes Sam to the concert, then Sam will take Bill to dinner.*  
Which of the following implies that this statement is true:
- (a) Sam takes Bill to dinner only if Bill takes Sam to the concert.
  - (b) Either Bill doesn't take Sam to the concert or Sam takes Bill to dinner.
  - (c) Bill takes Sam to the concert.
  - (d) Bill takes Sam to the concert and Sam takes Bill to dinner.
  - (e) Bill takes Sam to the concert and Sam doesn't take Bill to dinner.
  - (f) The concert is canceled.
  - (g) Sam doesn't attend the concert.
9. (**Question 2.30, p. 68**) Consider open sentences  $P(n) : 5n + 3$  is prime, and  $Q(n) : 7n + 1$  is prime, both over the domain  $\mathbb{N}$ .
- (a) State  $P(n) \Rightarrow Q(n)$  in words.
  - (b) State  $P(2) \Rightarrow Q(2)$  in words. Is this statement true or false?
  - (c) State  $P(6) \Rightarrow Q(6)$  in words. Is this statement true or false?
10. Consider the statement: *The fish are biting and there are no bugs, or the fish are not biting and there are bugs, or it is winter.* Write out the negation of the above in English. You should simplify your answer as much as possible, being sure of course that it is logically equivalent to the negation. Justify your answer.