1. (a) Prove that any number is congruent to the sum of its digits modulo 9.
   
   *Hint: consider a two-digit number ab first, and use the definition of congruence.*

   (b) Prove that any number is congruent to alternating sum of it digits mod 11. The alternating sum is: $a_0 - a_1 + a_2 - a_3 + a_4 - \cdots + (-1)^n a_n$, where $a_0$ is the last digit (the number of units).

2. (a) Prove that $n$ is even iff $n^3$ is even. (*Hint: The contrapositive would be very helpful in one direction.*)

   (b) Prove that $2^{1/3}$ is irrational. *Hint: Use proof by contradiction.*

3. Using induction, prove that the number written as 111...1 (with an even number of 1s) is always divisible by 11.

4. There are $n$ students in the class. Two volunteers are needed to help deliver a large box of exams to another building. Using induction, prove that there are $n(n-1)/2$ possible choices for the pair of volunteers.

5. Three friends - Adam, Bob and Cindy - are studying at UBC. One is in math, one is in physics, and one is in chemistry. If Adam is a mathematician, then Cindy is not a physicist. If Bob is not a physicist, then Adam is a mathematician. If Cindy is not a mathematician, then Bob is a chemist.

   Determine which friend is studying which subject.