1. **Exercises for Chapter 1: Question 1.2.** Let $S = \{-2, -1, 0, 1, 2, 3\}$. Describe each of the following sets as $\{x \in S : p(x)\}$, where $p(x)$ is some condition on $x$ (the task is to find the condition $p(x)$ for each of the sets below):

(a) $A = \{1, 2, 3\}$
(b) $B = \{0, 1, 2, 3\}$
(c) $C = \{-2, -1\}$
(d) $D = \{-2, 2, 3\}$.

**Solution:**

- $A = \{x \in S : x > 0\}$;
- $B = \{x \in S : x \geq 0\}$;
- $C = \{x \in S : x < 0\}$;
- $D = \{x \in S : |x| \geq 2\}$.

2. **Exercises for Chapter 1: Question 1.4.** Write each of the following sets by listing its elements within braces.

(a) $A = \{n \in \mathbb{Z} : -4 < n \leq 4\}$
(b) $B = \{n \in \mathbb{Z} : n^2 < 5\}$
(c) $C = \{n \in \mathbb{N} : n^3 < 100\}$
(d) $D = \{x \in \mathbb{R} : x^2 - x = 0\}$
(e) $E = \{x \in \mathbb{R} : x^2 + 1 = 0\}$.

**Solution:**

- $A = \{-3, -2, -1, 0, 1, 2, 3, 4\}$;
- $B = \{-2, -1, 0, 1, 2\}$;
- $C = \{1, 2, 3, 4\}$;
- $D = \{0, 1\}$;
- $E = \emptyset$.

3. **Exercises for Chapter 1: Question 1.10.** Give examples of three sets $A, B, C$ such that

(a) $A \subseteq B \subset C$
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(b) $A \in B$, $B \in C$, and $A \notin C$.
(c) $A \in B$ and $A \subseteq C$.

**Solution:** (a)
- $A = \{1\}$;
- $B = \{1\}$;
- $C = \{1, 2\}$.

(b)
- $A = \{1\}$;
- $B = \{\{1\}\}$;
- $C = \{\{\{1\}\}\}$.

(c)
- $A = \{1\}$;
- $B = \{\{1\}\}$;
- $C = \{1, 2\}$.

4. **Exercises for Chapter 1: Question 1.14.** Find $\mathcal{P}(A)$ and $|\mathcal{P}(A)|$ for $A = \{0, \emptyset, \{\emptyset\}\}$.

**Solution:** For the problem typed in the assignment, the solution goes as following.
- $\mathcal{P}(A) = \{\emptyset, \{0\}, \{\emptyset\}, \{\{\emptyset\}\}, \{0, \emptyset\}, \{0, \{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}, A\}; |\mathcal{P}(A)| = 8$.

5. **Exercises for Chapter 1: modified Question 1.20.** Determine whether the following statements are true or false (if false, give an example where it is not true; if true, prove):

(a) If $\{1\} \in \mathcal{P}(A)$, then $1 \in A$.
(b) If $\{1\} \in \mathcal{P}(A)$ then $\{1\} \notin A$.
(c) If four sets $A$, $B$, $C$, $D$ are subsets of $\{1, 2, 3\}$ such that $|A| = |B| = |C| = |D| = 2$, then at least two of these sets are equal.
(d) $A \subset \mathcal{P}(B)$ and $|A| = 2$, then $B$ has at least two elements.
Solution:

• (a) True. Otherwise, if $1 \notin A$, then $\{1\} \notin \mathcal{P}(A)$. This is a contradiction to the condition $\{1\} \in \mathcal{P}(A)$.

• (b) False. Counterexample: $A = \{1, \{1\}\}$. We see that $\{1\} \in \mathcal{P}(A) = \{\emptyset, \{1\}, \{\{1\}\}, \{1, \{1\}\}\}$, but $\{1\} \in A$.

• (c) True. There are only 3 possible subsets of $\{1, 2, 3\}$ with cardinality 2, i.e., $\{1, 2\}, \{1, 3\}, \{2, 3\}$. By the pigeonhole principle, for $A, B, C, \text{ and } D$, there must be at least two of them are equal.

• (d) True. We know the cardinality of $\mathcal{P}(B) = 2^{|B|}$ is a multiple of 2. Since $A \subset \mathcal{P}(B)$, it follows that $|\mathcal{P}(B)| > |A| = 2$. Hence, $2^{|B|} = |\mathcal{P}(B)| \geq 4 = 2^2$. This means that $|B| \geq 2$. 