Worksheet 9: gradients – solution to Problem 2

A hiker is walking up the trail on the mountain following the direction of the steepest ascent. The hiker’s speed is 3 km/hr. When the hiker is at point $A = (a, b, c)$ on the mountain, his compass is telling him that he walking directly Northwest; the slope of the trail is 30°.

(a) If the mountain is thought of the graph of the altitude function $z = f(x, y)$, find $|\nabla f|$ at the point $(a, b)$.

**Solution:** See the class notes for the solution of this part; we found:

$$\nabla f = \left \langle -\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right \rangle$$

(b) Find the velocity vector of the hiker at the point $A$.

**Solution:** Let $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ be the velocity of the hiker. Then we know the following:

- the vector made of its first two components is pointing NW, so $\langle v_1, v_2 \rangle = c\langle -1, 1 \rangle$ for some positive scalar $c$. This is because the hiker is going in the direction of the steepest ascent, which is NorthWest.
- The slope of the mountain in this direction is 30°. This means,

$$\frac{v_3}{\sqrt{v_1^2 + v_2^2}} = \tan(30°) = \frac{1}{\sqrt{3}}.$$ 

(this is just stating that the slope of the vector $\mathbf{v}$ with respect to the horizontal plane is the same as the slope of the mountain (which it has to be in order for the hiker to stay on the mountain)).
- We also are given the speed of the hiker, which is,

$$3 = |\mathbf{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}.$$

Putting it all together, we get: $\mathbf{v} = \langle -c, c, v_3 \rangle$; then from the second point, we get that $\frac{v_3}{c\sqrt{2}} = \frac{1}{\sqrt{3}}$, so $v_3 = c\frac{\sqrt{2}}{\sqrt{3}}$. Plugging it into the last point:

$$c^2 + c^2 + \frac{2}{3}c^2 = 3^2$$

1
Then \( \frac{8}{3}c^2 = 9 \), and \( c = \frac{3\sqrt{3}}{2\sqrt{2}} \).

**Answer:** \( v = \langle -\frac{3\sqrt{3}}{2\sqrt{2}}, \frac{3\sqrt{3}}{2\sqrt{2}}, \frac{3}{2} \rangle \).