How to remember quadric surfaces

In the equation, you can have linear terms (such as: \( z = x^2 + y^2 \)) or not.

If you do not: \( \frac{x^2}{a^2}, \frac{y^2}{b^2}, \frac{z^2}{c^2} \) and a constant are the only possible terms.

They can come with + or −.
The constant can be 0 or 1.

We get:

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad (\text{all } +, \text{ constant } \text{is } 1)
\]

- ellipsoid

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \quad (\text{one sheet})
\]

\[
\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \quad \{ \text{hyperboloids} \}
\]

\[
\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0 \quad \{ \text{two sheet} \}
\]

When the constant on the right is 0:

\[
\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0
\]

cone
Note: when right-hand side is 0, it doesn't matter whether you have one or two minus signs: just multiply the whole equation by (-1) and you can switch from one to the other. Also, when RHS = 0, cannot have all + signs:
\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0 \]  - defines only the point (0, 0, 0)
  - it is the only solution.

When you have linear term \( z \):
call it \( z \): get:
\[ \frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2} \]  \( \text{elliptic paraboloid} \)
\[ \frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2} \]  \( \text{hyperbolic paraboloid} \)

Note: on the plane, if you write \( y = x^2 \) (\( y \) is \textbf{not} squared), you get a parabola in space, when one of the variables is \textbf{not} squared, you get a \textbf{paraboloid}.

Now, \( x^2 + y^2 = 1 \) is an ellipse, \( x^2 - y^2 = 1 \) is a hyperbola. \( z \)-paraboloid is elliptic or hyperbolic accordingly.