
Let \( f(x, y) = \cos(x) + \sin(y) \).

(1) [3] Find the domain and range of \( f \).

**Solution:** Domain is \( \mathbb{R}^2 \) because all inputs \( (x, y) \) are possible. The range is \([-2, 2]\) (because clearly \( 0 \leq \cos(x) + \sin(y) \leq 2 \), and at the same time every value in this interval is attained since \( \cos(x) \) takes every value between \(-1\) and \(1\), and \( \sin(y) \) takes every value between \(-1\) and \(1\) and they are independent of each other).

(2) [3] Circle all the statements that apply to the level curves of \( f(x, y) \):

(a) They display a periodic pattern.

(b) There are no level curves outside of the square \(-2 \leq x \leq 2, -2 \leq y \leq 2\).

(c) The level curve \( f(x, y) = 0 \) is a union of straight lines.

**Solution:** We have: (a) and (c) true, (b) false. (a) is true because \( f(x + 2\pi k, y + 2\pi m) = f(x, y) \) for any pair of integers \((m, k)\). This means that if you take any point \((x, y)\) that lies on some level curve, and translate it by any multiple of \(2\pi\) in both \(x\)- and \(y\)-direction, you will wind up on the same level curve!

(b) is False: there is a level curve passing through every point in the domain, and since the domain is the whole plane, there are level curves everywhere.

(c) is True: consider the level curve \( f(x, y) = 0 \). We have the identities: \( \sin(x + \pi/2) = -\cos(x) \) and \( \sin(3\pi/2 - x) = -\cos(x) \), so the level curve \( f(x, y) = 0 \) is the union of all the lines \( y = x + \pi/2 + 2\pi k \), and the lines \( y = 3\pi/2 - x + 2\pi k \).

(3) [4] Give an equation for the plane tangent to the graph \( z = f(x, y) \) at the point \( P = (\pi/3, \pi/2, 3/2) \).

**Solution:** We compute: \( f_x = -\sin(x), f_y = \cos(y) \). Evaluate at \((\pi/3, \pi/2)\), get: \( f_x(\pi/3, \pi/2) = -\sqrt{3}/2, f_y(\pi/3, \pi/2) = 0 \). Then the equation of the tangent plane at \( P \) is:

\[
z = \frac{3}{2} - \frac{\sqrt{3}}{2} (x - \pi/3).
\]
(4) [2] Find a parametric equation of the tangent line at \( P \) to the curve of intersection of this graph with the plane \( x = \pi/3 \).

**Solution:** We need to find the direction vector of this tangent line. This vector has to lie in the \( yz \)-plane (since \( x \) is fixed), and its slope has to equal \( f_y(\pi/3, \pi/2) \), essentially by definition of the partial derivative. We can take the vector \( \langle 0, 1, f_y(\pi/3, \pi/2) \rangle \) (the only choice I have here is putting 1 as the \( y \)-component, which makes it easy to control the slope). This vector turns out to be \( \langle 0, 1, 0 \rangle \). We get the parametric equation of the tangent line:

\[
r(t) = \langle \pi/3, 1/2, 3/2 \rangle + t\langle 0, 1, 0 \rangle.
\]

(5) [3] Find the total differential of \( f(x, y) \) and use it to approximate \( f(\pi/3 - 0.1, \pi/2 + 0.2) \).

**Solution:** We already found the partial derivatives at \( (\pi/3, \pi/2) \), so the total differential at this point is:

\[
df = -\frac{\sqrt{3}}{2} dx + 0 \cdot dy.
\]

Using this, we get:

\[
f(\pi/4 - 0.1, \pi/2 + 0.2) \approx \frac{3}{2} - \frac{\sqrt{3}}{2} \cdot (-0.1).
\]