Quiz 2: Lines, planes, quadric surfaces. Section 201.

(1) Find an equation for the plane passing through the point \((1, 2, 3)\) and perpendicular to the line with the vector parametric equation

\[ \mathbf{r}(t) = \langle 11 + 5t, 17 + t, 3 \rangle. \]

Solution: The direction vector of the line is \(\langle 5, 1, 0 \rangle\) (get it by reading off the coefficients at \(t\)). To get a plane perpendicular to the line, we can simply use this vector as the normal vector for the plane. We get:

\[ 5(x - 1) + (y - 2) = 0. \]

(2) Find both symmetric and parametric equations for the line parallel to both planes \(x + z = 0\) and \(y + 2z = 15\) and passing through the point \((4, 5, 3)\).

Solution: Since we want the line to be parallel to both planes, its direction vector has to be perpendicular to the both normal vectors of these planes. We get: \(\mathbf{v} = \langle 1, 0, 1 \rangle \times \langle 0, 1, 2 \rangle = \langle -1, -2, 1 \rangle\). Answers: Parametric equation:

\[ x = 4 - t, \quad y = 5 - 2t, \quad z = 3 + t \]

Symmetric equation:

\[ \frac{x - 4}{-1} = \frac{y - 5}{-2} = \frac{z - 3}{1}. \]

(you could also simplify it, of course).
(3) Describe the intersection of the surface \(x^2 + 4y^2 + 9z^2 = 14\) with the plane \(x = y\).

**Solution:** The surface is an ellipsoid with centre at the origin, and we are intersecting it with the plane passing through the origin, so we get an ellipse. The coordinates of a point on this ellipse are \((x, x, z)\), where \(5x^2 + 9z^2 = 14\).