Suggested office hours:
M: 4-5
W: 2-3
F: 12:15-1:30

Last time:

unit vector: a vector of length 1.
Vectors

last time: \( \overrightarrow{V} \)

\[ \overrightarrow{V} B \]

A

Example: let \( \mathbf{A} = (1, 0, 2) \)
\[ \mathbf{B} = (3, 4, 5) \]

Let \( \overrightarrow{V} = \overrightarrow{AB} \)

Suppose we place \( \overrightarrow{V} \) so that its start is at \( \mathbf{O} \). Where does it end?

Answer \((2, 4, 3)\)
Connection between vectors and coordinates:

\[ A = (x_1, y_1, z_1) \]
\[ B = (x_2, y_2, z_2) \]
\[ \overrightarrow{AB} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle \]

\[ \text{displacement vector (from A to B)} \]

\[ \text{used to denote vectors.} \]

Points have coordinates, vectors have components.
In linear algebra, you write
\[ \mathbf{v} = [\mathbf{v}] \] (write components as a column)

Here, we write \( \mathbf{v} = \langle a, b, c \rangle \)

It is the same!

Operations:

1) addition.

\[ \mathbf{v}_1 + \mathbf{v}_2 : \]

Parallelogram rule:

\[ \mathbf{v}_1 + \mathbf{v}_2 : \]
\[ \vec{v}_1 = \langle a_1, b_1, c_1 \rangle \]
\[ \vec{v}_2 = \langle a_2, b_2, c_2 \rangle \]
\[ \vec{v}_1 + \vec{v}_2 = \langle a_1+a_2, b_1+b_2, c_1+c_2 \rangle. \]

2) **Scalar multiplication**

*a* - a real number

\[ \vec{v} \] - a vector

\[ a \vec{v} = \text{vector of length } |a| \cdot |\vec{v}| \]

parallel to \( \vec{v} \), and:

- points in the same direction as \( \vec{v} \) if \( a > 0 \)
- opposite of \( \vec{v} \) if \( a < 0 \)
- \( a \vec{v} = \vec{0} \) if \( a = 0 \).

zero vector
Now, have an easy way to go between a point \( P = (a, b, c) \) and vector \( \overrightarrow{OP} \):
\[
\overrightarrow{OP} = \langle a, b, c \rangle = a \cdot \mathbf{i} + b \cdot \mathbf{j} + c \cdot \mathbf{k}
\]

\[\text{resolution of } \langle a, b, c \rangle \text{ into components.}\]

The difference:
\[
\overrightarrow{V_1} - \overrightarrow{V_2} = \overrightarrow{V_1} + (-1) \cdot \overrightarrow{V_2}
\]

Algebraically,
\[
\overrightarrow{V_1} - \overrightarrow{V_2} = \langle a_1 - a_2, b_1 - b_2, c_1 - c_2 \rangle
\]
\[
\overrightarrow{V_1} = \langle a_1, b_1, c_1 \rangle
\]
\[
\overrightarrow{V_2} = \langle a_2, b_2, c_2 \rangle
\]

Scalar: \( k \) - real number
\[
\overrightarrow{k} = \langle a, b, c \rangle. \text{ Then } k \cdot \overrightarrow{V} = \langle ka, kb, kc \rangle
\]
Example: Excel spreadsheet thinks of data you enter in a column as a vector.

\( \mathbf{v}_1 \) = vector with 75 components = webwork scores / 100.

\( \mathbf{v}_2 \) = vector of midterm scores / 100.

\( \mathbf{v}_3 \) = vector of final exam / 100.

\( \mathbf{v}_4 \) = sum of 5 quizzes / 100.

Final marks: \( \mathbf{v} = \frac{1}{10} \mathbf{v}_1 + \frac{25}{100} \mathbf{v}_2 + \frac{50}{100} \mathbf{v}_3 + \frac{15}{100} \mathbf{v}_4 \)
The dot product:

Operation on vectors

\( \mathbf{v}_1 = \langle a_1, b_1, c_1 \rangle \)

\( \mathbf{v}_2 = \langle a_2, b_2, c_2 \rangle \)

\( \mathbf{v}_1 \cdot \mathbf{v}_2 = a_1 a_2 + b_1 b_2 + c_1 c_2 \)

\( \mathbf{v}_1 \cdot \mathbf{v}_2 = \text{number} \)

Why useful?

\( \mathbf{v} = \langle a, b, c \rangle \)

\( \mathbf{v} \cdot \mathbf{v} = a^2 + b^2 + c^2 = \| \mathbf{v} \|^2 \)
Properties:

\( \overline{v}_1, \overline{v}_2, \overline{v}_3 \) - vectors, \( k \) - scalar

1) \( \overline{v}_1 \cdot (\overline{v}_2 + \overline{v}_3) = \overline{v}_1 \cdot \overline{v}_2 + \overline{v}_1 \cdot \overline{v}_3 \)

2) \( (k \overline{v}_1) \cdot \overline{v}_2 = k \cdot (\overline{v}_1 \cdot \overline{v}_2) \)

... (please read 10.3)

---

3) \( \overline{v}_1 \cdot \overline{v}_2 \cdot \overline{v}_3 \) - scalar product

Not good b/c depends on parentheses:

\( \overline{v}_1 \cdot (\overline{v}_2 \cdot \overline{v}_3) \) parallel to \( \overline{v}_1 \)

Do not write \( \overline{v}_1 \cdot \overline{v}_2 \cdot \overline{v}_3 \)!