Today: Main topic: cross product

First, a word about work.

\[ \mathbf{F} \cdot \mathbf{d} \] displacement
\[ \mathbf{F} \] force applied

Then work of \( \mathbf{F} \) in displacing the object is

\[ W = \mathbf{F} \cdot \mathbf{d}. \]

Comment: Recall from last class:

\[ \begin{align*}
\uparrow & \quad \uparrow \\
\mathbf{v} & \quad \text{boat} \quad \mathbf{w} \\
\text{we decomposed the wind velocity} \quad \mathbf{w} & \quad \text{into} \\
\uparrow \text{proj} \mathbf{v} & \quad \text{components: parallel to} \quad \mathbf{v} \quad \text{and} \quad \uparrow \\
\text{only this component} & \quad \mathbf{w} - \text{proj} \mathbf{v} \mathbf{w} \\
\text{is doing nonzero work} & \quad \text{here work} \quad \mathbf{w} \cdot \mathbf{d} \quad \text{is} \quad 0 \\
\text{in displacing the boat} & \quad \text{b/c the} \quad \text{dot product} \\
& \quad \text{with displacement} \\
& \quad \text{is} \quad 0. 
\end{align*} \]
Another formula for work:

\[ W = (F \cdot \cos \alpha) \cdot d = \vec{F} \cdot \vec{d} \]

\[ \text{precisely the length of } \text{proj}_x \vec{F} \]

\[ = \text{comp}_x \vec{F} \]

\[ \text{depends on } \alpha < 90^\circ \]

\[ \alpha > 90^\circ \]
Cross product

\[ \vec{v} \times \vec{w} \]

Geometric: \( \vec{v} \times \vec{w} \)
is a vector perpendicular to the plane of \( \vec{v}, \vec{w} \)
or zero if \( \vec{v} \parallel \vec{w} \)
with direction determined by the right-hand rule.

length of \( \vec{v} \times \vec{w} \) is \( |\vec{v}| |\vec{w}| \sin \alpha \)

Notes: 1) if \( \vec{v} \parallel \vec{w} \) - parallel, then
\[ \vec{v} \times \vec{w} = \vec{0} \]

2) \[ \vec{w} \times \vec{v} = -\vec{v} \times \vec{w} \]

3) This only works in 3 dimensions!
Algebraically: \[ \overline{v} = \langle a_1, b_1, c_1 \rangle \]
\[ \overline{w} = \langle a_2, b_2, c_2 \rangle \]

Then \[ \overline{v} \times \overline{w} = \begin{vmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = i \left| b_1 c_2 \right| - j \left| b_2 c_1 \right| + k \left| a_1 b_2 \right| \]

\[ = (b_1 c_2 - c_1 b_2) \overline{i} - (a_1 c_2 - c_1 a_2) \overline{j} + (a_1 b_2 - b_1 a_2) \overline{k} \]

\[ \text{vector} = \langle b_1 c_2 - c_1 b_2, -(a_1 c_2 - c_1 a_2), a_1 b_2 - b_1 a_2 \rangle \]