Recall: solid: coordinate planes, the plane \( x+y=1, \ z=1-x^2 \)

Want: set up an integral over it as \( \iiint L \, dx \, dy \, dz \)

Last time: set it up \( \iiint_0^{1-x^2} \, dz \, dy \, dx \)

Intersection of \( z=1-x^2 \) with \( yz \)-plane

First: emphasize all "ribs" that you see

Next: think of more "ribs" -

where does the green surface meet the plane \( x+y=1 \)?
- parabolic cylinder meets a plane, intersection is a parabola?

What points does it contain?
\( (1,0,0), (0,1,1) \)
Now: problem 2. We want: \[ \iiint \, dxdydz \]

We have: \[ \iiint_0^{1-x} \, dxdydz \]

From the set-up we have:

\[ 0 \leq x \leq 1 \]
\[ 0 \leq y \leq 1-x \]
\[ 0 \leq z \leq 1-x^2 \]

\[ \text{max possible } z \]

Want: ? \leq z \leq ?

Rewrite boundaries for \( x,y \) & terms \( dz \).

\[ \text{min possible } z = 0 - \text{easy} \]

Max possible \( z \): \[ z \leq 1-x^2 \leq 1 \], so definitely \( z \leq 1 \).

Is \( z=1 \) possible? - Yes.

(Sketch: plug in \( x=0 \)).

So:

\[ \int_0^1 dz \]

our outside integral.
Now: want limits for \( y \) in terms of \( z \).

\( \text{cannot use } x!! \)

\[ 0 \leq y \leq 1-x \]

\[ \Rightarrow \]

\text{how big can this be?}

we also have:

\[ z \leq 1-x^2 \]

\[ x^2 \leq 1-z \]

\[ -\sqrt{1-z} \leq x \leq \sqrt{1-z} \text{ also } 0 \leq x. \]

\text{not relevant.}

So: the biggest \( 1-x \) is still \( 1 \):

\text{Got:} \quad 1-x \leq 1, \quad \star

\text{Looks like we got:} \quad \int_{0}^{1} \int_{0}^{1} dy \, dz

\text{Does this make any sense?}

we got these bounds when \( x=0 \).

Look at the cross-section by the plane \( x=0 \).

This agrees with the picture! \( \leftarrow \) Red squares (see sketch on p. 1)
Continue: \[ \int \int dy \, dz \]

What are the limits for \( x \)? (can use \( z, y \)).

We should have:

- \( 0 \leq y \leq 1 - x \)
- \( 0 \leq z \leq 1 - x^2 \)
- \( x \leq 1 - y \)
- \( x^2 \leq 1 - z \)
- \( x \leq \sqrt{1 - z} \)

Both have to hold.

(they are competing!)

Which one wins?

When is \( 1 - y = \sqrt{1 - z} \)? (all \( x \) positive by)

\[
\begin{align*}
1 - y &= \sqrt{1 - z} \\
(1 - y)^2 &= 1 - z \\
z &= 1 - (1 - y)^2
\end{align*}
\]

\( y = 1 - \sqrt{1 - z} \)

\[
\begin{array}{c}
\text{1} \\
\text{1} \\
\text{1} \\
\text{1}
\end{array}
\]

\( \sqrt{1 - z} \) is smaller
So the integral over the red square \((x_{1-x})\) breaks into two pieces:

\[
\iiint_{x_{1-x}} \; dx \; dy \; dz + \iiint_{y=1-x} \; dx \; dy \; dz
\]

For each \((y,z)\) in the square in the back, the integral \(dx\) is over the piece of the line parallel to the \(x\)-axis that lies inside the solid.

The line parallel to the \(x\)-axis exits the solid "through the roof" in the green part of the square; through the side wall: \(y=1-x\) in the black part.

Note: Setting up the integral in the other order:

\[
\iiint_{y=1-x} \; dy \; dx \; dz \quad \text{would be easier again:}
\]

\[
\iiint_{0 \leq x \leq \sqrt{1-z}} \; 1-x \; dy \; dx \; dz
\]

(the limits for \(z\) we figured out already: we also figured out \(0 \leq x \leq \sqrt{1-z}\) and the only constraint on \(y\) is)

\(0 \leq y \leq 1-x\)