Math 200

Lecture 1, January 7, 2020

Prof: Julia Gordon

office: Math 217

office hours: Friday 1-3
~~Thursday~~ Tuesday afternoon (To be confirmed)

website: common website
(linked from www.math.ubc.ca)

my homepage: www.math.ubc.ca/~gor

Canvas: - Webwork link
(go to Math 200 ALL course)
(not your section!)
and Piazza. - to discuss questions

Help: - office hours (or try appointment)
- Piazza! - 5 largest contributors get an extra point to the final grade!
- MLC

- Textbook: online (linked from common website).
- Tests: 4 tests in class. Dates will be posted tomorrow.

What is Math 200 about?

- situations that depend on many (more than one ☺) parameters.

Goal: find ways to simplify complex problems.

Main tool: linear algebra

Our main space: $\mathbb{R}^2$ - 2-dimensional
$\mathbb{R}^3$ - 3-dimensional.

First: vectors in space.
and coordinates.

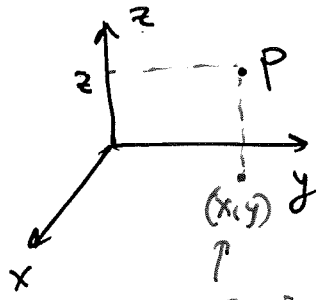
Examples of functions of many variables:

- temperature at a point in this room
  - function of x, y, z

- value of a portfolio composed of different stocks

- outcome of an experiment with many variable inputs

Coordinate system in $\mathbb{R}^3$



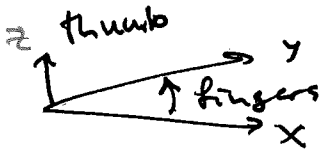
projection of P onto the xy-plane

- every point in space is determined by 3 numbers:

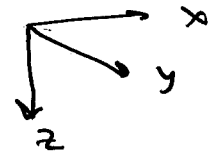
$$P = (x, y, z)$$

$(0,0,0)$ – the origin

<u>Convention</u>: <u>Right-hand rule</u>



from x to y

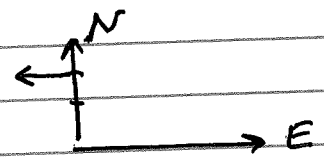


also ok.

Vectors: $\nearrow$ – has length and direction.

example: go 2 blocks North and then turn left and walk one block – defines a vector.

This vector would be denoted $\langle \underset{W}{-1}, \underset{N}{2} \rangle$ (if blocks are units)



← when talking about coordinates on a plane, **convention**: East = x-axis, North = y-axis

Formally: you can take any numbers and form a **vector** from them:

$\bar{v} = \langle a_1, \ldots, a_n \rangle$ ←

numbers, called **components** of a vector.

Vectors in $\mathbb{R}^2$ ← plane: have 2 components:

$\bar{v} = \langle a, b \rangle$ or $\langle a_1, a_2 \rangle$

vectors in $\mathbb{R}^3$ ← space: have 3 components:

$\bar{v} = \langle a, b, c \rangle$ or $\bar{v} = \langle a_1, a_2, a_3 \rangle$

Example spreadsheet of marks for Math 200.

column 1: Test 1 scores

$$\bar{v}_1 = \langle a_1, a_2, \text{---}, a_{125} \rangle$$

mark for test 1 of the student first alphabetically.

the mark for test 1 of the last (alphabetically) student.

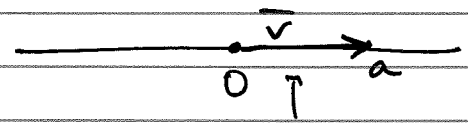
This vector lives in $\mathbb{R}^{125}$ ← 125-dimensional space!

Assignment: think of vectors keeping track of data you care for.

Key point about vectors:

1) There is a correspondence between points in $\mathbb{R}^n$ and ~~vec~~ vectors with n components $(n = 1, 2, 3, \ldots)$

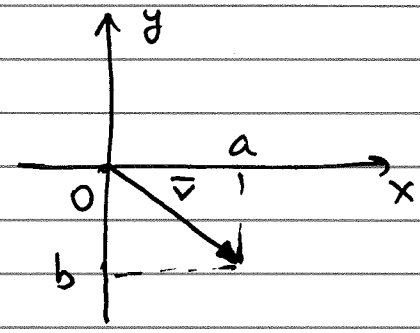
$n=1$:



vector $\bar{v} = \langle a \rangle$

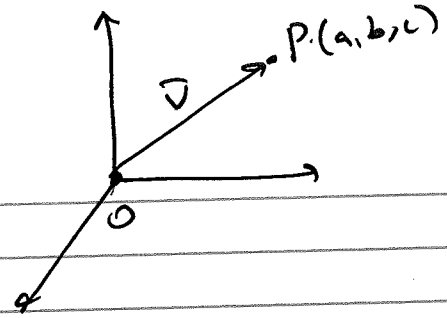
single component: a number

$n = 2$



$\bar{v} = \langle a, b \rangle$

in $\mathbb{R}^3$:



$\bar{v} = \langle a, b, c \rangle$

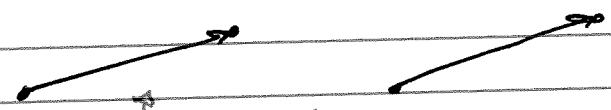
2) • The vector can have a start (tail) at <u>any</u> point.

if we start $\bar{v}$ at the origin, we get:

$\bar{v} = \overline{OP} = \langle a, b, c \rangle$ then $P = (a, b, c)$

↑ end

vector can be started <u>anywhere</u> :



— same vector

↑ same length, same direction.

<u>Question:</u> in linear algebra, vectors always start at $\bar{O}$

- a reasonable convention.
- good for making a coordinate definition:

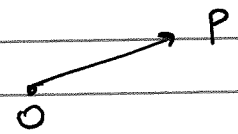
$$P = (a_1, \ldots, a_n)$$
$$\bar{v} = \overline{OP} = \langle a_1, \ldots, a_n \rangle$$

- <u>So far:</u> • points have coordinates, denoted by $(\, , \ldots , \,)$ ← round brackets.

• vectors have <u>components:</u> $\langle \, , \ldots \rangle$ – angular brackets

• points ↔ vectors

start $\bar{v}$ at the origin, take its end ← $\bar{v}$



Point P ⟶ $\overline{OP}$ ↗ origin

$(a, b, c) \longleftrightarrow \langle a, b, c \rangle$

Why have vectors?

- operations with vectors (which you cannot do on points)

- Addition
- scalar multiplication

- both can be defined algebraically or geometrically.

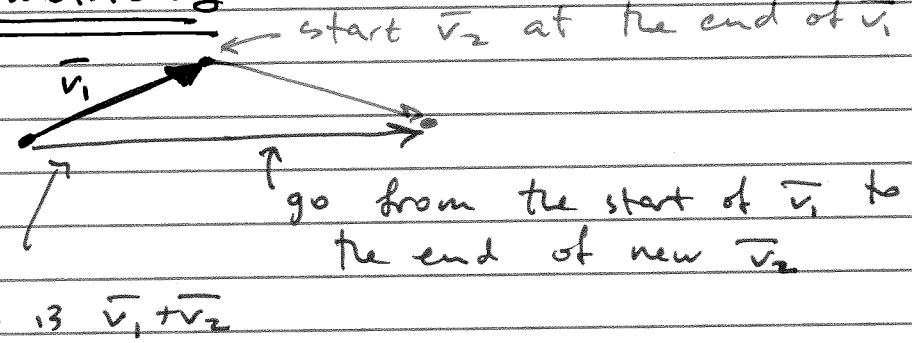
1) Algebraically

$\bar{v}_1 = \langle a_1, \ldots, a_n \rangle \qquad \bar{v}_2 = \langle b_1, \ldots, b_n \rangle$

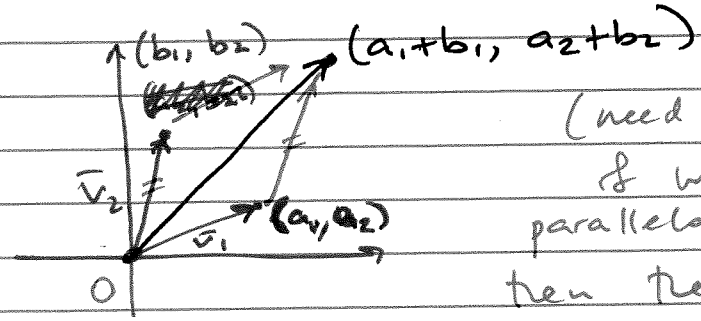
(in the same space)

Def $\bar{v}_1 + \bar{v}_2 \overset{def}{=} \langle a_1+b_1, \ldots, a_n+b_n \rangle$

2) Geometrically



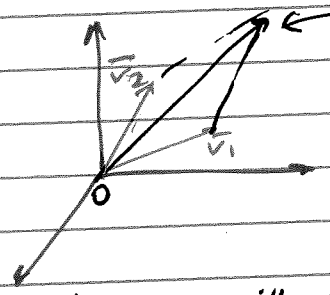
Theorem: These two defs give the same result:



(need to prove: if we make a parallelogram on $\bar{v}_1, \bar{v}_2$ then the end of the diagonal has coordinates (a_1+b_1, a_2+b_2))

This is good for:

- dealing with geometric problems algebraically



← convenient to know that we can simply add $\bar{v}_1$ and $\bar{v}_2$ to get the end of the diagonal of the parallelogram.

(later will use this to get a proof for law of cosines ← please recall it ☺)

One more operation:

- scalar multiplication:

$\bar{v} = \langle a_1, \ldots, a_n \rangle$, c – a number ($c \in \mathbb{R}$)

↑ c is a real number

Then
$$c \cdot \bar{v} = \langle ca_1, \ldots, ca_n \rangle$$

Geometrically: $c\bar{v}$ is a vector in the direction – same as $\bar{v}$ if $c > 0$
– opposite to $\bar{v}$ if $c < 0$

and length $= |c| \cdot \underbrace{\|\bar{v}\|}_{\text{length of } \bar{v}}$

if $c = 0$, $c \cdot \bar{v} = \bar{0}$ ← zero vector $= \langle 0, 0, \ldots, 0 \rangle$

(zero vector looks like a point)

Example: our marks spreadsheet.

let $\bar{v}_1$ = marks for test 1

$\bar{v}_2$ — test 2

...

$\bar{f}$ — marks for final exam (all out of 100)

$\bar{w}$ — marks for webwork

$\bar{t}$ = total final marks.

Spreadsheet operation at the end:

$$\bar{t} = 0.5\,\underbrace{\bar{f}}_{\text{out of } 100} + 0.1\left(\bar{v}_1 + \bar{v}_2 + \bar{v}_3 + \bar{v}_4\right) + 0.1 \cdot \bar{w}$$