Prof: Julia Gordon
office: Math 217
office hours: Friday 1-3
website: common website (linked from www.math.ubc.ca)
my homepage: www.math.ubc.ca/~jgor
Canvas: Webwork link (go to Math 200 ALL course) (not your section!)
and Piazza. to discuss questions
Help: office hours (or try appointment)
Piazza! 5 largest contributors get an extra point to the final grade!
MLC
Textbook: online (linked from common website).
Tests: 4 tests in class. Dates will be posted tomorrow.
What is Math 200 about?

- situations that depend on many (more than one) parameters.

Goal: find ways to simplify complex problems.

Main tool: linear algebra

Our main space: $\mathbb{R}^2$ - 2-dimensional
$\mathbb{R}^3$ - 3-dimensional.

First: vectors in space and coordinates.

Examples of functions of many variables:

- temperature at a point in this room - function of $x, y, z$

- value of a portfolio composed of different stocks

- outcome of an experiment with many variable inputs
Coordinate system in \( \mathbb{R}^3 \)

Every point in space is determined by 3 numbers: \( P = (x, y, z) \)

Projection of \( P \) onto the xy-plane

\( (0,0,0) \) - the origin

Convention: Right-hand rule

Thumb: from \( x \) to \( y \)

Fingers: from \( x \) to \( z \)

Also ok.
Vectors: - has length and direction.

Example: go 2 blocks North and then turn left and walk one block - defines a vector.

This vector would be denoted \( \langle -1, 2 \rangle \) (if blocks are units).

When talking about coordinates on a plane,

Convention: East = x-axis
North = y-axis

Formally: you can take any numbers and form a vector from them:

\[ \vec{v} = \langle a_1, a_2, \ldots, a_n \rangle \]

numbers, called components of a vector.
Vectors in $\mathbb{R}^2$ have 2 components:
$\mathbf{v} = \langle a_1, b \rangle$ or $\langle a_1, a_2 \rangle$

Vectors in $\mathbb{R}^3$ have 3 components:
$\mathbf{v} = \langle a_1, b, c \rangle$ or $\langle a_1, a_2, a_3 \rangle$

Example spreadsheet of marks for Math 200.

Column 1: Test 1 scores

$\mathbf{v}_i = \langle a_1, a_2, \ldots, a_{125} \rangle$

The mark for test 1 of the last (alphabetically) student.

This vector lives in $\mathbb{R}^{125} \subseteq 125$-dimensional space!

Assignment: think of vectors keeping track of data you care for.
Key point about vectors:

1) There is a correspondence between points in $\mathbb{R}^n$ and vectors with $n$ components ($n = 1, 2, 3, \ldots$)

$n = 1$:

Vector $\vec{v} = \langle a \rangle$

A single component: a number

$n = 2$:

Vector $\vec{v} = \langle a, b \rangle$
in $\mathbb{R}^3$:

$\vec{v} = \langle a, b, c \rangle$

2) The vector can have a start at any point. (tail)

If we start $\vec{v}$ at the origin, we get:

$\vec{v} = \overrightarrow{OP} = \langle a, b, c \rangle$ then $P = (a, b, c)$

Vector can be started anywhere:

same vector

same length, same direction.
Question: in linear algebra, vectors always start at 0
- a reasonable convention.
- good for making a coordinate definition:

\[ P = (a_1, \ldots, a_n) \]
\[ \overrightarrow{OP} = \langle a_1, \ldots, a_n \rangle \]

- So far: points have coordinates, denoted by \((\_\_\_\_), \_\_\_\_\_) \leftarrow \text{round brackets.}

- vectors have components: \((\_\_\_\_\_) \rightarrow \text{angular brackets}

- points \rightarrow \text{vectors}

start \( \overrightarrow{OP} \)

at the origin, take its end

\( \overrightarrow{OP} = (a_1, b, c) \rightarrow \langle a_1, b, c \rangle \)
Why have vectors?

- operations with vectors (which you cannot do on points)

- Addition
  - both can be defined algebraically or geometrically.

- Scalar multiplication

1) Algebraically

\[ \vec{v}_1 = \langle a_1, \ldots, a_n \rangle \quad \vec{v}_2 = \langle b_1, \ldots, b_n \rangle \]

( in the same space)

\[ \text{Def: } \vec{v}_1 + \vec{v}_2 \overset{\text{def}}{=} \langle a_1 + b_1, \ldots, a_n + b_n \rangle \]

2) Geometrically

\[ \text{start } \vec{v}_2 \text{ at the end of } \vec{v}_1 \]

\[ \text{go from the start of } \vec{v}_1 \text{ to the end of new } \vec{v}_2 \]

This is \( \vec{v}_1 + \vec{v}_2 \)

Theorem: These two def's give the same result:

\[ \text{(need to prove:)} \]

\[ \text{if we make a parallelogram on } \vec{v}_1, \vec{v}_2 \]

\[ \text{then the end of the diagonal has coordinates} \]

\[ \langle a_1 + b_1, a_2 + b_2 \rangle \]
This is good for:
- dealing with geometric problems algebraically
  
  Convenient to know that
  we can simply add $\vec{v}_1$ and $\vec{v}_2$ to get the end of the diagonal of the parallelogram.

  (Later will use this to get a proof for law of cosines ← please recall it 😊)

One more operation:

- scalar multiplication:
  $$ \vec{v} = \langle a_1, \ldots, a_n \rangle, \quad c \bullet - a \text{ number} $$
  \begin{align*}
  \text{(} \quad c \in \mathbb{R} \quad \text{)}
  \quad c \text{ is a real number}
  \end{align*}

  Then
  $$ c \cdot \vec{v} = \langle ca_1, \ldots, ca_n \rangle $$

Geometrically, $c \cdot \vec{v}$ is a vector in the same direction as $\vec{v}$ if $c > 0$
and opposite to $\vec{v}$ if $c < 0$

and length $= \left| c \right| \cdot \left| \vec{v} \right|$ 

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if $c = 0$, $c \cdot \vec{v} = \vec{0} ←$ zero vector

$\langle 0, 0, \ldots, 0 \rangle$

(zero vector looks like a point)
Example: our marks spreadsheet.

Let $\bar{v}_1 =$ marks for test 1
$\bar{v}_2 =$ test 2
\[ \vdots \]
$\bar{f} =$ marks for final exam (out of 100)
$\bar{w} =$ marks for webwork
$\bar{t} =$ total final marks.

Spreadsheet operation at the end:

$$\bar{t} = 0.5 \bar{f} + 0.1 (\bar{v}_1 + \bar{v}_2 + \bar{v}_3 + \bar{v}_4) + 0.1 \bar{w}$$

out of 100