

Office hrs: Tue 9:30 - 10:30

(Reminder) Thurs 4:15 - 5:45

Today: Quadric surfaces
(and cylinders)

• one equation in 3 variables defines a surface:

ex: $ax + by + cz + d = 0$ - plane

$x^2 + y^2 + (z-3)^2 = 4$ - sphere of radius 2
centre at $(0, 0, 3)$.

↑
quadric surface

↑
means: equation of degree 2.

General quadric surfaces:

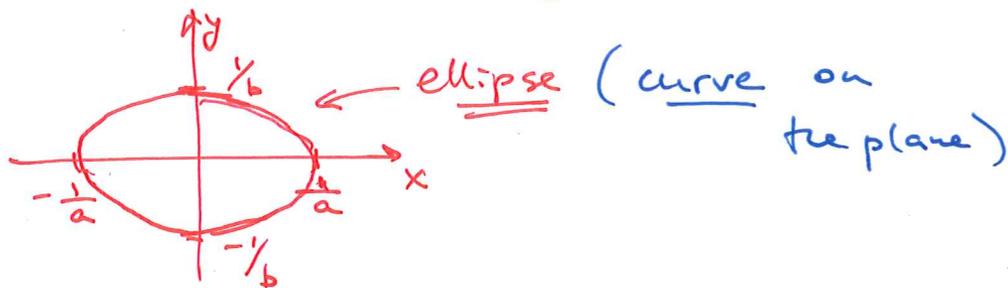
$$ax^2 + by^2 + cz^2 + \underbrace{dxy + e \cdot yz + f \cdot xz}_{\text{degree 2}} + \overbrace{gx + h \cdot y + l \cdot z}_{\text{linear part}} + m = 0.$$

↑
general degree 2
polynomial in 3 variables.

What does this kind of surface look like?

Note: The coefficients $\underline{a^2 x^2}$ & $\underline{b^2 y^2}$ make circles stretch into ellipses:

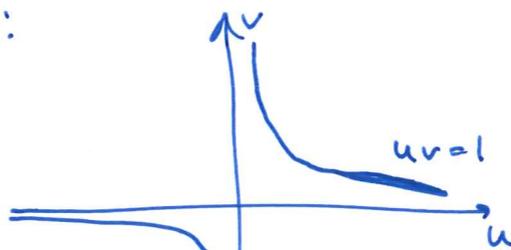
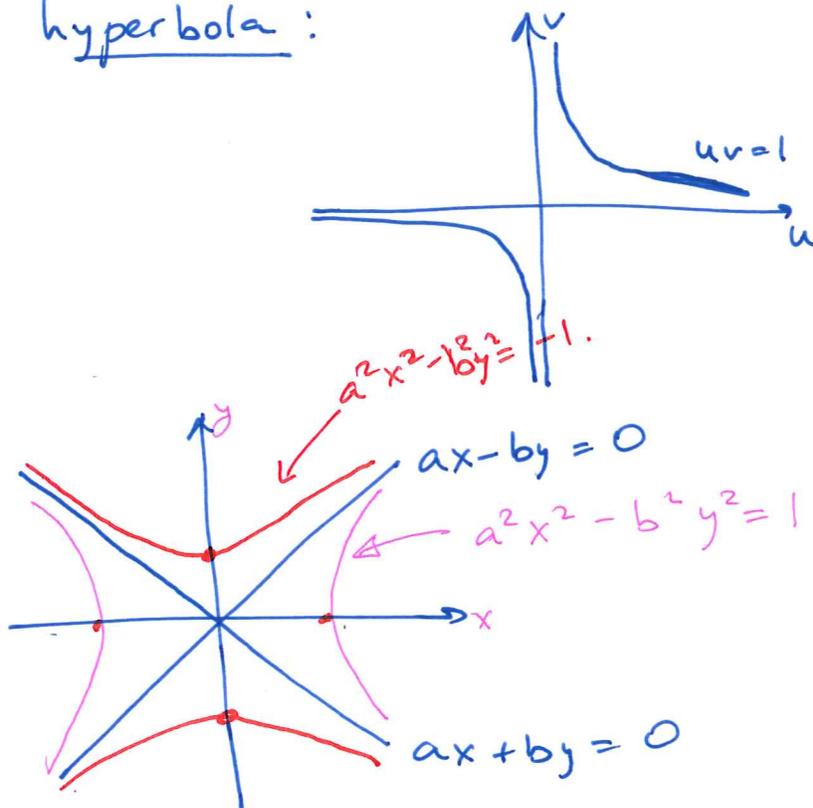
on the plane: $\underline{a^2 x^2 + b^2 y^2 = 1}$



hyperbola:

$$\underline{a^2 x^2 - b^2 y^2 = 1}$$

$$\underline{(ax - by)(ax + by) = 1.}$$



Fact: by algebraic manipulations (which correspond to linear coordinate changes), you can

reduce a general quadric surface to one of:

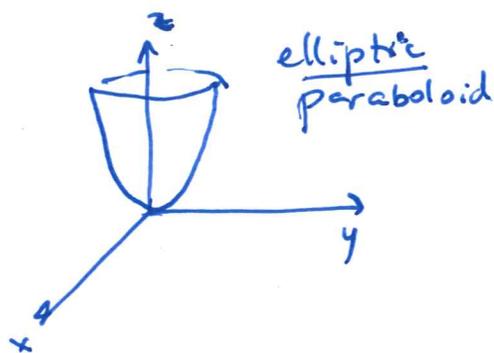
1) "degenerate": a plane or pair of planes

ex: $(x-y)(y-z)=0$ defines 2 planes:
 $x=y$ and $y=z$.

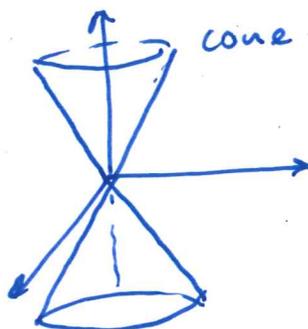


2) The simplest shapes:

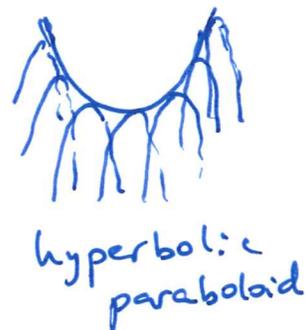
• $z = a^2x^2 + y^2b^2$



• $z^2 = a^2x^2 + y^2b^2$



• $z = a^2x^2 - y^2b^2$



• ellipsoid



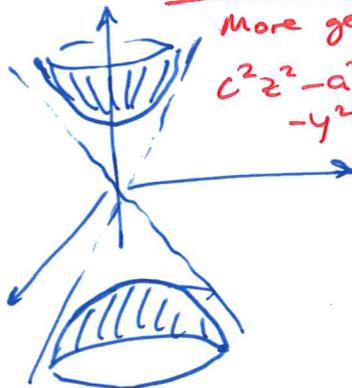
$\frac{z^2}{a^2} + \frac{y^2}{b^2} + \frac{x^2}{c^2} = 1$

$a, b, c > 0$

(sphere is a special case $a=b=c$)

• hyperboloids :

$z^2 - x^2 - y^2 = -1$

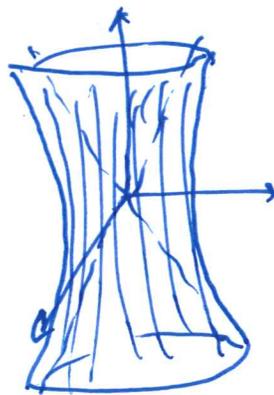


2-sheet hyperboloid

More generally:
 $c^2z^2 - a^2x^2 - y^2b^2 = -1$

$c^2z^2 - a^2x^2 - y^2b^2 = 1$

$z^2 - x^2 - y^2 = 1$



1-sheet hyperboloid