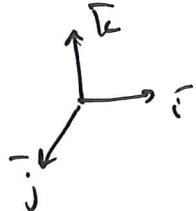


Office hours : The 9:30 - 10:30

Thurs : 4:15 - 5:45

Today: 1) Projections, Components \leftarrow in any number of dimensions.
2) Cross Product in \mathbb{R}^3 .

Recall: last class, discussed how $\bar{v} = \langle a, b, c \rangle$
can write



$$\bar{v} = a\bar{i} + b\bar{j} + c\bar{k}, \text{ and } a = \bar{v} \cdot \bar{i}$$

unit
vector
of x-axis

$$b = \bar{v} \cdot \bar{j}$$

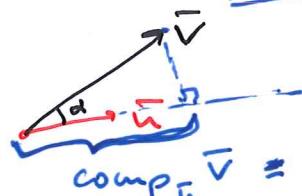
$$c = \bar{v} \cdot \bar{k}$$

Components of \bar{v}
along the axes.

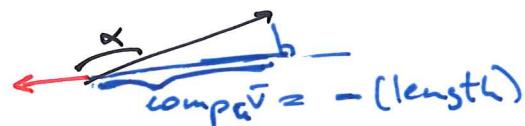
More generally:

let \bar{u} be any unit vector (means: $\|\bar{u}\| = 1$)
will use double

\bar{v} - any vector
Then the component of \bar{v} along \bar{u} is $\bar{v} \cdot \bar{u}$ $= \|\bar{v}\| \cos \theta$

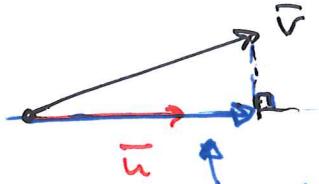


$\text{comp}_{\bar{u}} \bar{v} = \# \text{ this length.}$



(sometimes called scalar component)

More useful: Vector projection



want this vector - vector projection of \vec{v} onto \vec{u}
(or vector component)

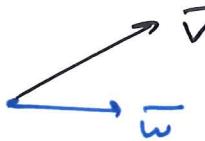
$$\text{proj}_{\vec{u}} \vec{v} = (\text{comp}_{\vec{u}} \vec{v}) \cdot \vec{u} = (\vec{v} \cdot \vec{u}) \vec{u}$$

vector, parallel to \vec{u}
so it should be $c \cdot \vec{u}$
where c is a scalar.
 $c = \text{comp}_{\vec{u}} \vec{v}$

dot product,
a scalar
vector parallel to \vec{u}

This works for unit \vec{u} .

What if we want to project \vec{v} onto a vector that's not unit?



Step 1: The vector that you are projecting onto is made into a unit vector:

$$\text{Let } \vec{u} = \frac{\vec{w}}{\|\vec{w}\|} = \underbrace{\frac{1}{\|\vec{w}\|}}_{\text{unit vector}} \cdot \vec{w}$$

in the same direction as \vec{w} .

Step 2: use the earlier formula for projection:

$$\bar{u} = \frac{\bar{w}}{\|\bar{w}\|}$$

$$\text{proj}_{\bar{w}} \bar{v} = \text{proj}_{\bar{u}} \bar{v} = (\bar{v} \cdot \bar{u}) \cdot \bar{u} = \left(\bar{v} \cdot \frac{\bar{w}}{\|\bar{w}\|} \right) \cdot \frac{\bar{w}}{\|\bar{w}\|}$$

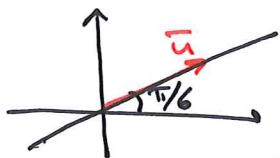
$$\boxed{\text{proj}_{\bar{w}} \bar{v} = \left(\bar{v} \cdot \frac{\bar{w}}{\|\bar{w}\|} \right) \cdot \frac{\bar{w}}{\|\bar{w}\|} = \frac{\bar{v} \cdot \bar{w}}{\|\bar{w}\|^2} \cdot \bar{w}}$$

scalar.

Reality check: vector parallel to \bar{w} .

Worksheet 1: vectors and forces

1. What is the general form of a vector \mathbf{v} on the plane that points upward and forms the angle of $\pi/6$ with the positive direction of the x -axis?

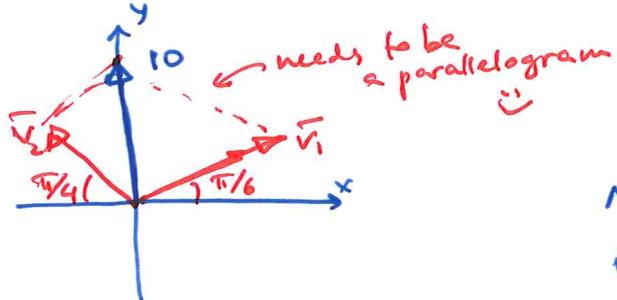


Let \mathbf{u} be the unit vector in the specified direction.

$$\text{Then } \mathbf{u} = \left\langle \cos \frac{\pi}{6}, \sin \frac{\pi}{6} \right\rangle = \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$$

Answer: $\mathbf{v} = c \cdot \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$ for $c > 0$.

2. Find two vectors \mathbf{v}_1 and \mathbf{v}_2 on the plane such that \mathbf{v}_1 forms the angle of $\pi/6$ with the positive direction of the x -axis, \mathbf{v}_2 forms the angle of $3\pi/4$ with the positive direction of the x -axis, and $\mathbf{v}_1 + \mathbf{v}_2 = \langle 0, 10 \rangle$.



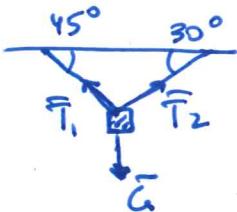
From Problem 1: $\mathbf{v}_1 = c_1 \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$

$$\mathbf{v}_2 = c_2 \left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$$

Need to find: c_1, c_2 such that
 $\mathbf{v}_1 + \mathbf{v}_2 = \langle 0, 10 \rangle$.

Get a system of equations: see next page.

3. A small block of mass 1kg hangs on two chains such that one chain form the angle of 45° and the other the angle of 30° with the horizontal. Find the forces that act on the block.



$$\bar{T}_1 + \bar{T}_2 + \bar{G} = \bar{0} \quad (\text{block is not moving})$$

$$\text{so } \bar{T}_1 + \bar{T}_2 = -\bar{G}$$

Get Problem 2 except $-\bar{G} = \langle 0, 9.8 \rangle$

'depends on units.'

Replace "10" with 9.8 in Problem 2, you are done.

($9.8 \text{ N} = \text{Force of } 1 \text{ kg acting on 1 kg}$).

Problem 2, continued :

$$c_1 \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle + c_2 \left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle = \langle 0, 10 \rangle$$

$$\begin{cases} \frac{\sqrt{3}}{2} c_1 - \frac{1}{\sqrt{2}} c_2 = 0 \\ \frac{1}{2} c_1 + \frac{1}{\sqrt{2}} c_2 = 10 \end{cases}$$

Rename: $\frac{1}{\sqrt{2}} c_2 = a$

w.th th.3 notation:

$$\begin{cases} \frac{\sqrt{3}}{2} c_1 = a \\ \frac{1}{2} c_1 = 10 - a \end{cases} \Leftrightarrow \begin{cases} c_1 = \frac{2a}{\sqrt{3}} \\ \frac{a}{\sqrt{3}} = 10 - a \end{cases}$$

$$\Rightarrow a = \frac{10}{1+\sqrt{3}} \quad \text{so} \quad c_1 = \frac{20}{\sqrt{3}(1+\sqrt{3})}$$

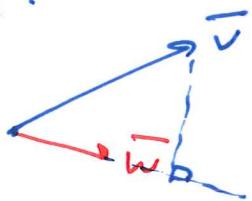
Answer: $\tilde{v}_1 = \left\langle \frac{10\sqrt{3}}{1+\sqrt{3}}, \frac{10}{1+\sqrt{3}} \right\rangle$

$$\tilde{v}_2 = \left\langle -\frac{10\sqrt{3}}{1+\sqrt{3}}, \frac{10\sqrt{3}}{1+\sqrt{3}} \right\rangle$$

In problems 2 and 3, the difficulty was that we had to decompose \vec{v} into components whose directions were specified but not perpendicular to each other.

Easier problem: Decompose \vec{v} as a sum of a component along \vec{w} and perp. to \vec{w} .

Recipe:



use vector projection!

$$\vec{v} = \text{proj}_{\vec{w}} \vec{v} + (\vec{v} - \text{proj}_{\vec{w}} \vec{v})$$

perp. to \vec{w}
(automatically)

Example: Fly is flying with velocity $\vec{v} = \langle 4, 5, 6 \rangle$
(some units)

wind is blowing with velocity $\langle 1, 2, 3 \rangle = \vec{w}$

Decompose the velocity of the fly into components along the wind and orthogonal to the wind.

Solution: $\text{proj}_{\vec{w}} \vec{v} = (\vec{v} \cdot \vec{u}) \cdot \vec{u} = \frac{\langle 4, 5, 6 \rangle \cdot \langle 1, 2, 3 \rangle}{14} \cdot \langle 1, 2, 3 \rangle$

Note

$$\|\vec{w}\| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$$

$$\vec{u} = \frac{1}{\sqrt{14}} \langle 1, 2, 3 \rangle - \begin{matrix} \text{unit vector} \\ \text{in the dir. of } \vec{w} \end{matrix}$$

$$= \frac{4+5 \cdot 2 + 6 \cdot 3}{14} \langle 1, 2, 3 \rangle$$

$$= \left\langle \frac{32}{14}, \frac{64}{14}, \frac{96}{14} \right\rangle$$

Orthogonal component:

$$\vec{v} - \text{proj}_{\vec{w}} \vec{v} = \langle 4, 5, 6 \rangle - \left\langle \frac{32}{14}, \frac{64}{14}, \frac{96}{14} \right\rangle = \dots$$

Component of the wind that doesn't matter to the fly

see next page for correct interpretation of these vectors.

Correction: correct interpretation:

$\left\langle \frac{32}{14}, \frac{64}{14}, \frac{96}{14} \right\rangle$ - the component of the velocity of the fly along the wind.

If I want the component of the wind that's helping the fly, should take:

$$\text{proj}_{\vec{v}} \vec{w}$$

if write

$$\vec{w} = \underbrace{\text{proj}_{\vec{v}} \vec{w}}_{\text{component of the wind helping the fly}} + (\vec{w} - \text{proj}_{\vec{v}} \vec{w})$$

$\underbrace{(\vec{w} - \text{proj}_{\vec{v}} \vec{w})}_{\text{component of the wind irrelevant to the fly.}}$

New topic

- Cross product : only works in 3 dimensions!

operation on vectors, called cross product

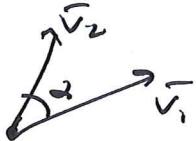
takes 2 vectors, $\vec{v}_1 = \langle a_1, b_1, c_1 \rangle$

$$\vec{v}_2 = \langle a_2, b_2, c_2 \rangle$$

makes a vector $\vec{v}_1 \times \vec{v}_2$

Geometric: this vector has length

$$\|\vec{v}_1 \times \vec{v}_2\| = \|\vec{v}_1\| \cdot \|\vec{v}_2\| \cdot \sin \alpha$$



- perpendicular to both \vec{v}_1, \vec{v}_2
direction is determined by
the right-hand rule
(it matters whether \vec{v}_1 , or \vec{v}_2
goes first!)

- $(\vec{v}_1 \times \vec{v}_2 = 0 \text{ if } \vec{v}_1 \parallel \vec{v}_2)$

Homework: Read the algebraic def'n.