Integration:

- Integral of \( f(x,y) \)

Recall: we had definite integral

\[
\int_a^b f(x) \, dx = \text{a number}
\]

if \( f(x) \geq 0 \) on \([a,b]\)

then it was equal to area under the graph.

Definition used:

Riemann sums.

But to compute, we used antiderivatives:

If \( F(x) = \int f(x) \, dx \) (which means, \( F'(x) = f \))

Then

\[
\int_a^b f(x) \, dx = F(b) - F(a) \quad \text{Fundamental Theorem of Calculus.}
\]
In $2^{nd}$, we will not have indefinite integral!
Definite integral will be over a region in $\mathbb{R}^2$.

![Diagram of a region D in the plane]

- closed (bounded) domain

we will define:

$$\iint_D f(x, y) \, dA$$

**Notation:**

$$\int_a^b f(x) \, dx$$

parts of the same notation

Now:

$$\iint_D f(x, y) \, dA$$

"double integral"

"will respect to area"

parts of the same notation

(for functions of 2 variables).
Easiest to define:

integrals over rectangles.

\[ R = [a,b] \times [c,d] \] - rectangle

Define \( \int_R \frac{f(x,y)}{R} \) some function on \( R \).

Riemann sums:

- subdivide \( R \) into small pieces
- take a sample from each piece
- pick a point \((x_{ij}^*, y_{ij}^*)\)

\[ 1 \leq i \leq n \]
\[ 1 \leq j \leq m \]

Write a sum:

\[ \sum_{1 \leq i \leq n} \sum_{1 \leq j \leq m} f(x_{ij}^*, y_{ij}^*) (\Delta x \Delta y) \]

area of our small rectangle

sample value on our rectangle
If \( f(x,y) \) is continuous,

the point: As the subdivision gets finer,

there is a limit.

This limit is called the \( \iiint f(x,y) \, dA \) over \( R \).

To compute, sum along \( x \),

then add results up along \( y \):

\[
\int_{a}^{b} \left( \int_{c}^{d} f(x,y) \, dx \right) \, dy
\]

\( \rightarrow \) iterated integral.

Think of \( y \) as a fixed constant.

\( \rightarrow \) should be a function of \( y \).

The inner integral computes this.