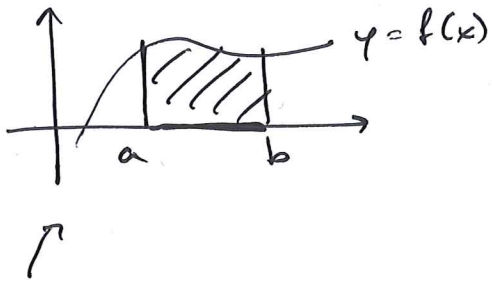
Integration:

- Integral of $f(x,y)$

Recall: we had definite integral



$\int_a^b f(x)\,dx$ – a number

if $f(x) \geq 0$ on $[a,b]$

then it was equal to area under the graph.

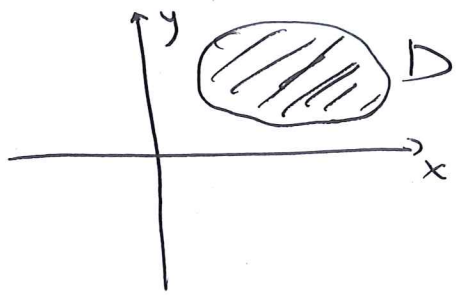
definition used Riemann sums.

but to compute, we used antiderivatives:

if $F(x) = \int f(x)\,dx$ (which means, $F'(x) = f$)

then $\int_a^b f(x)\,dx = F(b) - F(a)$ – Fundam. Thm of Calculus.

In 2^d, we will not have indefinite integral!
Definite integral will be over a region in $\mathbb{R}^2$



- closed (bounded) domain

we will define

$$\iint_D f(x,y)\, dA$$

<u>Notation</u>: $\displaystyle\int_a^b f(x)\, dx$

parts of the same notation

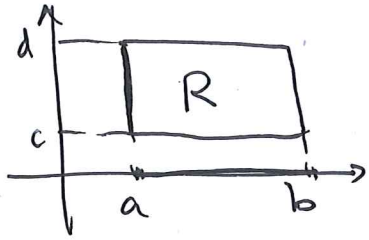
<u>Now</u>: $\displaystyle\iint_D f(x,y)\, dA$ ← "with respect to area"

"double integral"

parts of the same notation

(for functions of 2 variables).

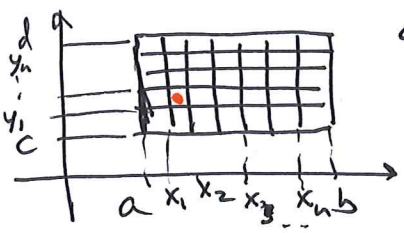
Easiest to define : integrals over rectangles.



$R = [a,b] \times [c,d]$ - rectangle

Define $\iint_R \underbrace{f(x,y)}_{\text{some function on } R.} \, dA$

Riemann sums:

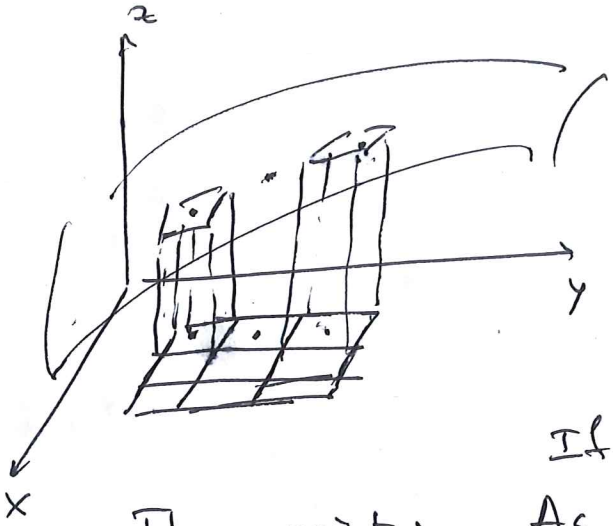


subdivide R into small pieces
take a sample from each piece
pick a point
(x_{ij}^*, y_{ij}^*)
$1 \le i \le n$
$1 \le j \le m$

Write a sum:

$$\sum_{\substack{1 \le i \le n \\ 1 \le j \le m}} \underbrace{f(x_{ij}^*, y_{ij}^*)}_{\text{sample value on our rectangle}} \boxed{\Delta x_i \, \Delta y_j}$$

area of our small rectangle



$z = f(x,y)$

approximately building "skyscrapers" up to graph of $f(x,y)$

If $f(x,y)$ is continuous,

<u>The point</u>: As the subdivision gets finer, there is a limit.

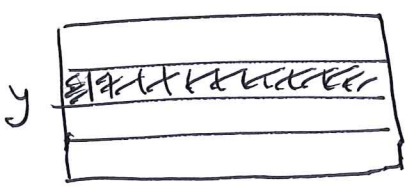
This limit is called the $\boxed{\iint_R f(x,y)\,dA}$ // volume under the graph if $f(x,y) \geq 0$.

<u>To compute</u>: sum along x then add results up along y:

$$\int_c^d \underbrace{\left(\int_a^b f(x,y)\,dx \right)}_{\text{think of } y \text{ as a fixed constant.}} dy$$

← <u>iterated</u> integral.

← should be a function of y.



← inner integral computes this.