

## Today: Geometric meaning of the gradient.

Key point:  $\bar{u}$  - unit vector in the domain of a function  $f$ .  $\leftarrow$  (assume smooth)

Then  $D_{\bar{u}} f = \bar{\nabla} f \cdot \bar{u}$

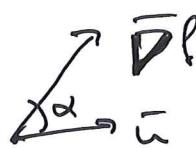
$\uparrow$   
directional derivative  
in the direction of  $\bar{u}$

"vector of partial derivatives"  $\left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \dots \right\rangle$   
all variables of  $f$ .

Consequences:  $D_{\bar{u}} f = 0 \iff \bar{u} \perp \bar{\nabla} f$

- $D_{\bar{u}} f$  is maximal among all directions if  $\bar{u} \uparrow \bar{\nabla} f$  (here  $D_{\bar{u}} f = \|\bar{\nabla} f\|$ )
- $D_{\bar{u}} f$  is the largest negative if  $\bar{u} \uparrow \bar{\nabla} f$  (here  $D_{\bar{u}} f = -\|\bar{\nabla} f\|$ )

(because  $\bar{\nabla} f \cdot \bar{u} = \|\bar{\nabla} f\| \cdot \underbrace{\|\bar{u}\|}_{=1} \cdot \cos \alpha$ )



## Geometric interpretation:

- $D_u f = 0$  when  $u \perp \nabla f$ :

$\bar{u}$  tangent  
to the

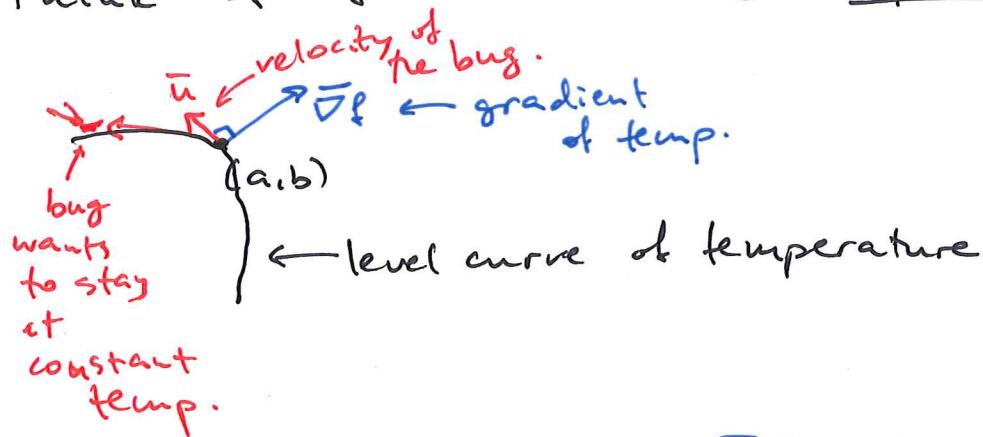
level curves of  $f$  ( $f(x,y)$ )

or level surface ( $f(x,y,z)$ )

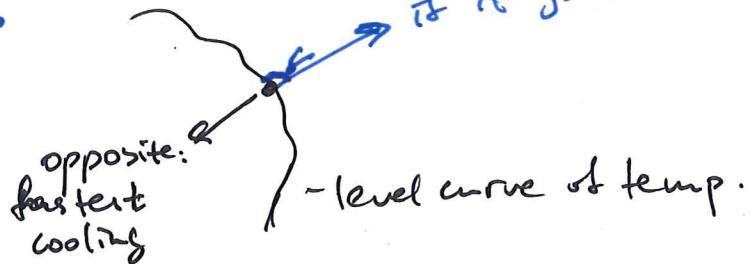
so:  $\nabla f$  is  
normal to  
level curves/  
level surfaces.

To make sense of this: do NOT think about graph of  $f$ !

Think of  $f$  as measuring temperature.



- if it goes along  $\nabla f$ , it will maximize the increase in temp.



Gradient points in the direction of the fastest increase of our function and is perpendicular to its level curves/surfaces.

- Next class:
- finish gradients  
(bring worksheet, think about it!)
  - start critical points / max min problems.

Note about gradients and "fastest ascent":

"gradient points in the direction of the steepest ascent":

- only makes sense when talking about a graph:

$$z = f(x, y)$$



At a point on the mountain, direction of the steepest ascent: a vector in  $3^d$ .

gradient of  $f$  is a vector with two components  
(a vector on the map, compass direction)

want to go in the direction of the steepest ascent,  
the xy-projection of your velocity is ~~the~~  
parallel to the gradient of  $f$ .

And the slope (the  $z$ -component) is  
determined by the shape of the mountain  
(proportional to  $\|\nabla f\|$ )

Please think about this at  
home and bring it on Thursday.

Worksheet 8: gradients

1. Let  $f(x, y)$  be a function of two variables. Find the gradient of the function  $F(x, y, z) = z - f(x, y)$ .

2. A hiker is walking up the trail on the mountain following the direction of the steepest ascent. The hiker's speed is  $3\text{km/hr}$ . When the hiker is at point  $A = (a, b, c)$  on the mountain, his compass is telling him that he is walking directly Northwest; the slope of the trail is  $30^\circ$ .

- (a) If the mountain is thought of the graph of the altitude function  $z = f(x, y)$ , find  $|\nabla f|$  at the point  $(a, b)$ .

- (b) Find the velocity vector of the hiker at the point  $A$ .