First Name: _____ Last Name: _____

Student-No: _____ Section: _____

Grade:

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Short answer question

1. [2 marks] Each part is worth 1 mark. Please write your answers in the boxes. Only your answers in the boxes will be marked (no partial credit).

   (a) If we start with the integral $\displaystyle\int_0^8 \frac{e^{\sqrt{t+1}}}{\sqrt{t+1}}\,dt$ and make the substitution $u = \sqrt{t+1}$, we obtain a new definite integral of the form $\displaystyle\int_a^b (\text{some function of } u)\,du$. What are a and b?

   Answer: $a = 1$, $b = 3$

   Solution: When $t = 0$, we have $u = \sqrt{0+1} = \sqrt{1} = 1$. When $t = 8$, we have $u = \sqrt{8+1} = \sqrt{9} = 3$. (An answer written in the form $\int_1^3$ or $[1, 3]$ is also acceptable.)

   (b) Calculate $\displaystyle\int_0^8 \frac{e^{\sqrt{t+1}}}{\sqrt{t+1}}\,dt$. Simplify your answer completely.

   Answer: $2e^3 - 2e$ or $2e(e^2 - 1)$

   Solution: Using the substitution $u = \sqrt{t+1}$, we see that $du = \dfrac{1}{2\sqrt{t+1}}\,dt$ and so

$$\int_0^8 \frac{e^{\sqrt{t+1}}}{\sqrt{t+1}}\,dt = \int_0^8 2e^{\sqrt{t+1}}\frac{1}{2\sqrt{t+1}}\,dt = \int_1^3 2e^u\,du = 2e^u\Big|_1^3 = 2e^3 - 2e.$$

   Alternatively, one can calculate the indefinite integral

$$\int \frac{e^{\sqrt{t+1}}}{\sqrt{t+1}}\,dt = \int 2e^{\sqrt{t+1}}\frac{1}{2\sqrt{t+1}}\,dt = \int 2e^u\,du = 2e^u + C = 2e^{\sqrt{t+1}} + C$$

   and then use the Fundamental Theorem of Calculus to obtain

$$\int_0^8 \frac{e^{\sqrt{t+1}}}{\sqrt{t+1}}\,dt = 2e^{\sqrt{t+1}}\Big|_0^8 = 2e^{\sqrt{8+1}} - 2e^{\sqrt{0+1}} = 2e^3 - 2e.$$

Long answer question—you must show your work

2. [4 marks] Calculate the area of the region beneath the curve $y = 2x^2 \log x$, above the x-axis, and to the left of the line $x = e$.

Solution: Noting that the curve $y = 2x^2 \log x$ intersects the x-axis when $x = 1$, the area is given by

$$\int_1^e 2x^2 \log x \, dx.$$

We use integration by parts, choosing $u = \log x$ and $dv = 2x^2 \, dx$, and setting $du = \dfrac{1}{x} \, dx$ and $v = \dfrac{2x^3}{3}$, so that

$$\int_1^e 2x^2 \log x \, dx = \frac{2x^3 \log x}{3}\bigg|_1^e - \int_1^e \frac{2x^2}{3} \, dx = \frac{2x^3 \log x}{3}\bigg|_1^e - \frac{2x^3}{9}\bigg|_1^e$$
$$= \left(\frac{2e^3}{3} - 0\right) - \left(\frac{2e^3}{9} - \frac{2}{9}\right) = \frac{4e^3 + 2}{9}.$$

Marking scheme:

- 1 mark for setting up the correct integral
- 1 mark for reasonable choices for u, dv, du and v
- 1 mark for applying the integration by parts formula correctly
- 1 mark for a final answer consistent with earlier work

Long answer question—you must show your work

3. [4 marks] Let R be the region enclosed by the y-axis and the curve $x = y^2 + y$. Calculate (using "horizontal slices") the volume obtained by rotating R about the y-axis.

Solution: The curve $x = y^2 + y$ is a parabola opening rightward, with y-intercepts $(0, 0)$ and $(0, -1)$. We slice the volume along the y-axis, obtaining the volume

$$\begin{aligned}\int_{-1}^{0} \pi \left(y^2 + y\right)^2 \, dy &= \int_{-1}^{0} \pi \left(y^4 + 2y^3 + y^2\right) \, dy \\ &= \pi \left(\frac{y^5}{5} + \frac{2y^4}{4} + \frac{y^3}{3}\right)\Bigg|_{-1}^{0} \\ &= 0 - \pi \left(\frac{(-1)^5}{5} + \frac{2(-1)^4}{4} + \frac{(-1)^3}{3}\right) = \frac{\pi}{30}.\end{aligned}$$

Marking scheme:

- 1 mark for the correct interval of integration on the volume integral
- 1 mark for the correct integrand in the volume integral
- 1 mark for the correct antiderivative of the chosen integrand
- 1 mark for a final answer consistent with earlier work