

## Math 101—Midterm Exam #2, Practice Midterm B

Duration: 50 minutes

\_\_\_\_\_  
Surname (Last Name)

\_\_\_\_\_  
Given Name

\_\_\_\_\_  
Student Number

**Do not open this test until instructed to do so!** This exam should have 8 pages, including this cover sheet. No textbooks, notes, calculators, or other aids are allowed; phones, pencil cases, and other extraneous items cannot be on your desk. Turn off cell phones and anything that could make noise during the exam.

Problems 1–4 are short-answer questions: put a box around your final answer, but no credit will be given for the answer without the correct accompanying work. Problems 5–7 are long-answer: give complete arguments and explanations for all your calculations—answers without justifications will not be marked. Continue on the back of the page if you run out of space.

### UBC rules governing examinations:

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
  - (a) speaking or communicating with other examination candidates, unless otherwise authorized;
  - (b) purposely exposing written papers to the view of other examination candidates or imaging devices;
  - (c) purposely viewing the written papers of other examination candidates;
  - (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
  - (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)—(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

Problem	Out of	Score	Problem	Out of	Score
1	6		5	8	
2	6		6	8	
3	6		7	8	
4	3		<b>Total</b>	45	

Problems 1–4 are short-answer questions: put a box around your final answer, but no credit will be given for the answer without the correct accompanying work.

1a. **[3 pts]** Determine whether the following sequences converge or diverge. If they converge, find the limit

(a)  $a_n = \frac{\sin n}{\sqrt{n+1}}$

(b)  $b_n = \ln(n+1) - \ln n$

(c)  $c_n = \cos\left(\frac{\pi n}{2}\right)$

1b. **[3 pts]** Determine whether the improper integral

$$\int_1^{\infty} \frac{4 + \sin x}{\sqrt{x - 1/2}} dx$$

converges or not. If it converges, find its value.

Problems 1–4 are short-answer questions: put a box around your final answer, but no credit will be given for the answer without the correct accompanying work.

2a. **[3 pts]** Use Simpson's Rule with  $n = 4$  to approximate the value of the integral

$$\int_{-3}^5 (2x^2 + 3x) dx.$$

2b. **[3 pts]** Evaluate  $\int_0^{\pi/4} \tan^3 x \sec^2 x dx$ . Simplify your answer fully.

Problems 1–4 are short-answer questions: put a box around your final answer, but no credit will be given for the answer without the correct accompanying work.

3a. **[3 pts]** Find the general antiderivative of  $f(x) = x^2 \sin(\pi x)$ .

3b. **[3 pts]** Evaluate  $\int_1^2 \frac{x^2}{\sqrt{x^2 - 1}} dx$ .

Problems 1–4 are short-answer questions: put a box around your final answer, but no credit will be given for the answer without the correct accompanying work.

4. [3 pts] Evaluate  $\int_0^4 \frac{x-1}{x^2-4x-5} dx$ .

Problems 5–7 are long-answer: give complete arguments and explanations for all your calculations—answers without justifications will not be marked.

5.

- (a) **[4 pts]** A lamina with density  $\rho = 3$  is in the shape of the region under the graph of  $y = e^{2x}$  between  $x = -1$  and  $x = 1$ . Find  $M_y$ , the moment with respect to the  $y$ -axis, of this lamina.

- (b) **[4 pts]** Evaluate the integral

$$\int (x + 1)\sqrt{x^2 + 2x + 2} dx.$$

6.

(a) **[4 pts]** Use integration to calculate the area of the ellipse

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

Hint: exploit the symmetry of the ellipse with respect to both axes.

6b. **[4 pts]** Determine whether the integral

$$\int_0^{e^3} \ln x \, dx$$

converges or diverges. If it converges, find its value.

7.

(a) **[8 pts]** Find the solution to the differential equation

$$\frac{dy}{dx} = \frac{y}{x^4 + x^2}$$

which satisfies the initial condition  $y(1) = 1/e$ .