Math 101—Midterm Exam #1, Practice Midterm A
Duration: 50 minutes

Name: ______________________ Student Number: ______________________

Do not open this test until instructed to do so! This exam should have 8 pages, including this cover sheet. No textbooks, notes, calculators, or other aids are allowed; phones, pencil cases, and other extraneous items cannot be on your desk. Turn off cell phones and anything that could make noise during the exam.

Problems 1–4 are short-answer questions: put a box around your final answer, but no credit will be given for the answer without the correct accompanying work. Problems 5–7 are long-answer: give complete arguments and explanations for all your calculations—answers without justifications will not be marked. Continue on the back of the page if you run out of space.

UBC rules governing examinations:

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
   (a) speaking or communicating with other examination candidates, unless otherwise authorized;
   (b) purposely exposing written papers to the view of other examination candidates or imaging devices;
   (c) purposely viewing the written papers of other examination candidates;
   (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
   (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)—(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

Problem	**Out of**	**Score**	**Problem**	**Out of**	**Score**
1	6		5	8	
2	6		6	8	
3	6		7	8	
4	3		**Total**	45	

Problems 1–4 are short-answer questions: put a box around your final answer, but no credit will be given for the answer without the correct accompanying work.

1a. **[3 pts]** Write $\displaystyle\int_2^7 \ln(x^2-1)\,dx$ as a limit of Riemann sums with right endpoints. Do not evaluate the Riemann sums or the limit.

1b. **[3 pts]** Express $\displaystyle\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n}\left(\frac{i^2}{n^2}-\frac{3i}{n}\right)$ as a definite integral, and evaluate the integral.

Problems 1–4 are short-answer questions: put a box around your final answer, but no credit will be given for the answer without the correct accompanying work.

2a. **[3 pts]** Evaluate the definite integral $\displaystyle\int_{-3}^{3} 3x^5\sqrt{9-x^2}\,dx.$

2b. **[3 pts]** Evaluate the indefinite integral $\displaystyle\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx.$

Problems 1–4 are short-answer questions: put a box around your final answer, but no credit will be given for the answer without the correct accompanying work.

3a. **[3 pts]** Evaluate the indefinite integral $y = \displaystyle\int \frac{x^2}{1+x^6}\,dx$.

3b. **[3 pts]** Find the slope of the tangent line to the curve $y = \displaystyle\int_{-2}^{x^3} e^{-t^2}\,dt$ at $x = -1$.

Problems 1–4 are short-answer questions: put a box around your final answer, but no credit will be given for the answer without the correct accompanying work.

4. **[3 pts]** A particle moves along a line so that its velocity at time t is given $v(t) = \cos(\pi t)$ (measured in metres per second). Find the total distance traveled during the first 3 seconds.

Problems 5–7 are long-answer: give complete arguments and explanations for all your calculations—answers without justifications will not be marked.

5a. **[2 pts]** Sketch the region enclosed by the graphs of $y = e^x$ and $y = \ln x$, the x- and y-axes, and the line $x = e$. Label the curves and the points of intersection.

5b. **[6 pts]** Evaluate the area of the region from part a. (Hint: Cut the region in two using the line $y = 1$.)

6a. **[2 pts]** Sketch the region bounded by the curves $y = 1 + \sin x$ and $y = \cos x$ and the lines $x = 0$ and $x = \frac{\pi}{2}$. Label the curves and the points of intersection.

6b. **[6 pts]** Find the volume of the solid obtained by rotating the region from part a about the x-axis. (Hint: $\sin^2 x - \cos^2 x = -\cos(2x)$.)

7. **[8 pts]** A tank in the shape of a hemispherical bowl of radius 3 m, with an outlet that rises 2 m above its top (see the diagram below), is full of water. Using the fact that the density of water is $1000\,\mathrm{kg/m^3}$, find the work (in joules) required to pump all the water out of the outlet. You may use the value $g = 9.8\,\mathrm{m/s^2}$ for the acceleration due to gravity. *You do not need to simplify your answer, but you must completely evaluate any integral that arises.*