(1) Find the number of ways that 2016 can be written in the form $\sum_{i \geq 0} a_i 2^i$, where the $a_i$ are allowed to take the values 0, 1, 2, or 3.

(2) Let $C_1$, $C_2$, and $C_3$ be three circles of radius 1 in the plane, no two of which are tangent to each other, that all pass through a point $P$. Each pair of circles intersects in a second point besides $P$; call these three other points of intersection $Q$, $R$, and $S$. Now draw three new circles $D_1$, $D_2$, and $D_3$ of radius 1, centered at the three points $Q$, $R$, and $S$. Show that $D_1$, $D_2$, and $D_3$ all intersect in a common point.

(3) Let $S$ be the set of all $2016 \times 2016$ matrices all of whose entries are 1 or $-1$. Let $M = \max \{ \det(A) : A \in S \}$ be the largest determinant of any matrix in $S$. Prove that $M$ is a multiple of $2^{2015}$.

(4) Let $S$ be a set of 2017 distinct positive real numbers. Prove that there is an ordering $S = \{ a_0, a_1, \ldots, a_{2016} \}$ of the numbers in $S$ such that the polynomial

$$a_{2016}x^{2016} + a_{2015}x^{2015} + \cdots + a_2x^2 + a_1x + a_0$$

has no real roots.

(5) Find a positive integer $B$ with the following property: there are exactly 2011 positive integers $A < B$ such that

$$\text{lcm}(A, B) + \gcd(A, B) = A + B.$$ 

(6) Determine

$$\lim_{x \to 0^+} \left( x \left( x \left( \cdots \left( x\left( x \cdots \left( x \right) \cdots \right) \right) \right) \right) \right).$$

(7) Let $n$ be a positive integer, and suppose that $n^n$ is a $K$-digit number (when written in base 10 as usual). Suppose further that $|K - 10^{100}| \leq 100$. How many digits does $n$ itself have?

(8) Suppose that $F$ is a finite set of points in the plane, containing at least three points, with the following property: anytime a line is drawn through two points of $F$, that line contains a third point of $F$ (possibly other points of $F$ as well, but at least three in total). Prove that all of the points in $F$ lie on a single line.