QP1. Suppose that $y$ is a real number such that $y + 1/y$ is an integer. Prove that $y^{2016} + 1/y^{2016}$ is also an integer.

QP2. Define a sequence of integers $\{x_n\}$ recursively by $x_1 = 2$ and

$$x_{n+1} = \left\lfloor \frac{3x_n}{2} \right\rfloor \quad \text{for } n \geq 1.$$ 

Prove that the sequence $\{x_n\}$ contains infinitely many even integers and infinitely many odd integers. (Here, $\lfloor y \rfloor$ denotes the greatest integer that is less than or equal to $y$.)

QP3. A polynomial $F(x)$ (with real coefficients) is called balanced if it has at least one positive root and at least one negative root. Prove that if $F(x)$ is a balanced polynomial, then

$$F(F(F(\cdots F(x)\cdots))))$$

is also a balanced polynomial.