1 mark each. Total 5 marks.

1. How many solutions does the system

\[
\begin{bmatrix}
0 & 1 & 3 & 2 \\
1 & 1 & 5 & 4
\end{bmatrix}
\]

have?

(a) 0
(b) 1
(c) 2
(d) 10
(e) infinitely many

Answer: (e). Reason: After exchanging the rows, we can see that the first two columns contain the pivots. The third column is a non-pivotal column, whose corresponding variable must be a parameter that generate infinitely many solutions.

2. How many solutions does the system

\[
\begin{bmatrix}
■ & * & * & * & * \\
0 & 0 & ■ & * & * \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

have, if ■ is a non-zero scalar and * is any scalar?

(a) 0
(b) 1
(c) 2
(d) 10
(e) infinitely many

Answer: (a). Reason: The last column, which is the constant vector, contains a pivot. This means the last equation must read 0 = ■ which is absurd.

3. How many solutions does the system

\[
\begin{bmatrix}
1 & 1 & 3 & 10 \\
0 & 2 & 6 & 100 \\
0 & 0 & 7 & 1000
\end{bmatrix}
\]

have?

(a) 0
(b) 1
(c) 2
(d) 10
(e) infinitely many

Answer: (b). Reason: All columns except the constant vector contains a pivot.

4. For which value of h is

\[
\begin{bmatrix}
5 \\
h
\end{bmatrix}
\]

in the plane spanned by

\[
\begin{bmatrix}
1 \\
0 \\
1
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
2 \\
1 \\
4
\end{bmatrix}
\]?

(a) 1
(b) 3
(c) 5
(d) 7
(e) 9

Answer (d). Reason: Solution exists only when \( h - 7 = 0 \), or \( h = 7 \).

5. Given three non-zero vectors \( \mathbf{u}, \mathbf{v}, \mathbf{w} \) in \( \mathbb{R}^3 \), which of the following must be true?

(a) \( \mathbf{w} \) lies in \( \text{Span}\{\mathbf{u}, \mathbf{v}\} \).
(b) \( \text{Span}\{\mathbf{u}\} \) is a line passing through the origin.
(c) \( \text{Span}\{\mathbf{u}, \mathbf{v}\} \) is a plane passing through the origin.
(d) \( \text{Span}\{\mathbf{u}, \mathbf{v}, \mathbf{w}\} \) is \( \mathbb{R}^3 \).
Answer: (b). Reason: (a) is not true when $w$ does not on the plane spanned by $\{u, v\}$ or not. (b) is true for a non-zero vector $u$, as shown in the notes. (c) is not true when $v$ is a linear combination of $u$, which means it is a multiple of $u$. (d) is not true when $w$ is a linear combination of $\{u, v\}$, which means it is on the plane spanned by $\{u, v\}$. 