Goals of Midterm 2

At this moment, you should be able to achieve the following goals. (The page numbers refer to those of the pdf handouts.)

Warning: Due to the unexpected high mean of Midterm 1, Midterm 2 will be much harder!!

1. About matrix inverse
   - the meaning (Sec 2.2, p.4,10),
   - how to compute it (Sec 2.2, p.7,10), and
   - the properties (Sec 2.3, p.2).

2. The Invertible Matrix Theorem: all consequences when $A$ is invertible. (p.10 of Sec 2.3 and p.12 of Sec 2.5-2.6 Part 3),

3. Understand the concept of a subspace (p.3-7 of Sec 2.5-2.6, part 1) and a basis of a subspace (p.8-9 of Sec 2.5-2.6, part 1).

4. Given a matrix $A$,
   - How to find a basis for $\text{Col } A$ (extracting linearly independent columns, see p.3-5 of Sec 2.5-2.6, part 2)
   - How the columns depend on the basis vectors
     (by reading the non-pivotal columns of the REF, see p.3-5 of Sec 2.5-2.6, part 2)
   - How to find a basis for $\text{Nul } A$
     (parametric vector form of the homogeneous equation, see p.11 of Sec 2.5-2.6, part 2)
   - Relation of the dimensions of $\text{Col } A$ and $\text{Nul } A$
     (the rank theorem, see p.3 of Sec 2.5-2.6, part 3)

5. Coordinate vector: given $x$ in $\mathbb{R}^n$ and a basis $B$ for $\mathbb{R}^n$.
   - The meaning of $[x]_B$, the coordinate vector of $x$ relative to the basis $B$ (p.4-6,9 of Sec 2.5-2.6, part 3),
   - how to transit between $x$ and $[x]_B$ (p.5,6 of Sec 4.4).

6. Coordinate matrix: given $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ and bases $B$ for $\mathbb{R}^n$, $C$ for $\mathbb{R}^m$.
   - The meaning of the matrix $[T]_C^B$ (p.9-10 of Sec 4.4) and how to compute it (p.11 of Sec 4.4)
   - how to transit between $T$ and $[T]_C^B$ by diagram chasing (p.12 of Sec 4.4).

7. Understand the properties of determinants (summarized in p.6 of Sec 3.1-3.2, part 1, or p.3-4 of part 2). Especially,
   - the linear property (summarized in p.25 of Sec 3.1-3.2, part 1)
   - the change under row operation (summarized in p.5 of Sec 3.1-3.2, part 2)

8. Compute a determinant using
   - Gaussian elimination (p.5 of Sec 3.1-3.2, part 2).
   - cofactor expansion (p.7 of Sec 3.1-3.2, part 3).
Practice exercises

• Sec 2.2: Practice problem 2, Exercises 1,3,5,8,9a-d,10,13, 17, 29, 31 (use the algorithm I showed in class), 35
  (The following questions require the big Invertible Matrix Theorem) 21, 22, 23, 24

• Sec 2.3: Practice problems 1,2,3, Exercises 1,3,5, 11, 12, 13, 14,
  (The following questions require the big Invertible Matrix Theorem) 15, 17, 18, 19, 20, 21,
  24, 35, 36, 39

• Sec 2.5 (Sec 2.8 in the 4th ed.): 5,7,9,10, 15,17,19,20, 21b,c,d,e, 22b,c, 24,26, 32,33,34,35,36

• Sec 2.6 (Sec 2.9 in the 4th ed.): 1,5,7,9,11,16, 17 a,c,d,e, 18 a,b,c,d,e, 19,21, 22,25

• Sec 4.4 (Sec 5.4 in the 4th ed.): 1,3,11,27,28,29,31
  (To approach the following questions, read the hints in Question 19) 19,20,21,24,25

• Sec 3.1: 1,9,11,13,19,21,23,37,38,39a, 40b.

• Sec 3.2: 1,3,5,7,15,17,18,19,21,22,24, 27a,c,d, 28a,b,29,39.
  (For the following questions, see the last two pages of the handout on Sec. 3.1-3.2, Part 1)
  31,32,33,34,35,36

You may check the solutions at:
The Invertible Matrix Theorem: all consequences when \( A \) is invertible.

(p.10 of Sec 2.3 and p.12 of Sec 2.5-2.6 Part 3),

- j. k. existence of the matrix \( A^{-1} \)
- As a linear transformation
  - f. \( A \) is one-to one, or
  - i. \( A \) is onto,
- The uniqueness of solution
  - g. For every \( b \), the system \( Ax = b \) has only 1 solution.
  - d. The homogeneous system \( Ax = 0 \) has only trivial solution.
- Reduced echelon form
  - b. The RREF of \( A \) is \( I \).
  - c. The REF of \( A \) has \( n \) pivots.
- The column vectors of \( A \)
  - e. are linearly independent,
  - h. span \( \mathbb{R}^n \)
  - m. form a basis of \( \mathbb{R}^n \)
- The column space of \( A \)
  - n. is the whole \( \mathbb{R}^n \),
  - o. p. \( \text{dim col } A = \text{rank } A = n \)
- The null space of \( A \)
  - q. is the zero subspace \( 0 \),
  - r. \( \text{dim nul } A = \text{nullity } A = 0 \).
- l. Transpose \( A^T \) is also invertible
  \( \Rightarrow \) The above statements about columns are also true about rows.
- t. \( \det A \neq 0 \).