Exercises from the textbook (Grinstead & Snell 2nd revised edition)

1. Section 2.2 (p.72) Ex. 6

2. If $X$ is an exponential random variable with parameter $\lambda$, and $c > 0$, show that $cX$ is exponential with parameter $\lambda/c$.

3. If $X$ is an exponential random variable with parameter $\lambda$, show that for any $k \geq 1$, $E[X^k] = \frac{k!}{\lambda^k}$.
   (hint: recall that $\int_0^{+\infty} y^k e^{-y} dy = k!$).

4. Section 5.2 Ex. 10 (p.220)

5. Section 5.2 Ex. 29

6. Section 6.3 Ex. 3 (p.278)

7. You arrive at a bus stop at 10am knowing the bus will arrive at time uniformly distributed between 10am and 10:30am. What is the probability you will wait at least 10mn? If no bus has arrived at 10:15am, what is the probability you will have to wait at least an additional 10mn?

8. Let $Z_1, Z_2, Z_3, \ldots$ be independent copies of a random variable $Z$ of expected value $\mu$ and variance $\sigma^2 > 0$. Let $X_n = \frac{Z_1 + \cdots + Z_n}{n}$. Using Chebyshev’s inequality, show that for any $a \geq 0$ with $a < 1/2$, the random variable $Y_n$ defined as $n^a (X_n - \mu)$ converges to 0 (it means you have to show that for $x < 0$, $P(Y_n \leq x)$ converges to 0, and for $x > 0$, $P(Y_n \leq x)$ converges to 1).