Exercises from the textbook (Grinstead & Snell 2nd revised edition)

1. Section 3.2 (p.115) Ex. 20

2. Multinomial coefficients
For \( m \geq 1 \) and \( n_1, \ldots, n_m \) positive integers, with the notation \( n \) for the sum \( n_1 + \cdots + n_m \), denote by \( C[n_1, \ldots, n_m] \) the number of ways to place \( n \) balls (of labels 1, 2, \ldots, \( n \)) into \( m \) urns \( U_1, \ldots, U_m \), such that there are \( n_1 \) balls falling into \( U_1 \), \ldots, \( n_m \) balls falling into \( U_m \).

a) Show that for \( m = 2 \), \( C[n_1, n_2] = \binom{n}{n_1} \).

b) For \( m = 3 \), show that

\[
C[n_1, n_2, n_3] = \binom{n}{n_1} \binom{n-n_1}{n_2},
\]

and show that this simplifies to

\[
C[n_1, n_2, n_3] = \frac{n!}{n_1!n_2!n_3!}.
\]

c) Show that \( C[n_1, n_2, n_3] \) also gives the numbers of words of length \( n \) with letters \( a, b \) or \( c \), such that \( a \) appears \( n_1 \) times, \( b \) appears \( n_2 \) times, \( c \) appears \( n_3 \) times.

d) Compute how many words can be formed from the letters of “assesses”.

e) Generalize the approach of (b) to show that

\[
C[n_1, \ldots, n_m] = \frac{n!}{n_1! \cdots n_m!}.
\]

3. Section 4.1 (p.150) Ex. 3

4. Section 4.1 Ex. 9 (assuming each newborn has equal chances to be boy or girl)

5. Section 4.1 Ex. 15

6. Section 4.1 Ex. 18

7. Section 4.1 Ex. 22

8. Section 4.1 Ex. 24