Exercises from the textbook (Grinstead & Snell 2nd revised edition)

1. Section 3.1 (p.88) Ex. 5

2. Section 3.1 Ex. 6

3. Section 3.1 Ex. 7

4. Section 3.1 Ex. 8

5. Section 3.1 Ex. 12

6. Section 3.1 Ex. 15

7. Section 3.1 Ex. 17

8. For \( k \geq 0 \) and \( n \geq 1 \) denote by \( w_n^{(k)} \) the number of permutations \(^{1}\) in \( S_n \) that have exactly \( k \) fixed points. Note that \( w_n^{(0)} \) is the number \( w_n \) of \( n \)-derangements, which (as we have seen) satisfies \( w_n / n! \rightarrow 1/e \) as \( n \rightarrow \infty \). Show that

\[
 w_n^{(k)} = \binom{n}{k} \cdot w_{n-k}.
\]

Deduce from this formula that for any fixed \( k \geq 0 \), the probability that a random permutation in \( S_n \) (under the uniform distribution) has exactly \( k \) fixed points converges to the constant \( \frac{1}{e \cdot k!} \) as \( n \rightarrow \infty \).

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\(^1\) \( S_n \) denotes the set of all \( n! \) permutations on the \( n \)-element set \( \{1, 2, \ldots, n\} \).