This week and next week: trigonometric and exponential functions.

ex. Serfling baseline model describing (cyclical) death due to pneumonia or influenza in 4-week time intervals:

\[ y(t) = 300.5 + 2.1t + 97.6 \cos \left( \frac{2\pi t}{13} - 2.67 \right) \]

In general, trigonometric functions like \( \cos \left( \frac{2\pi t}{13} - 2.67 \right) \) are used to model periodic phenomena.
Definitions. We will define the functions \( \sin(\theta) \), \( \cos(\theta) \), and \( \tan(\theta) \) three different ways.

1. The unit circle. Consider the circle of radius 1 centered on the origin. The point \((\cos(\theta), \sin(\theta))\) on the unit circle corresponds to the angle \(\theta\) in the standard position.
We will define $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$ (where this is defined).

2. The graphs.

$y = \sin(\theta)$

$y = \cos(\theta)$
[Note: Special values of \( \sin(\theta) \) and \( \cos(\theta) \).]
Note: the identity \( \sin^2(\theta) + \cos^2(\theta) = 1 \) is immediately apparent from the unit circle.

**Exercise**: Sin in the rest of the unit circle.

**3. Special Triangles**

Via similar triangles:

\[
\frac{\sin(\theta)}{H} = \frac{A}{H} = \frac{A}{A/H} = 0
\]

\[
\frac{\cos(\theta)}{H} = \frac{H}{H} = \frac{H}{0/H} = A/H
\]

\[
\tan(\theta) = \frac{0}{A/H} = 0
\]
What about the derivatives?

\[ y = \sin(\theta) \]

\[ y = \frac{1}{d\theta} \sin(\theta) \quad \text{— looks like } \cos(\theta) ! \]
EXERCISE: sketch the graph of \( \cos(\theta) \) and its derivative (without cheating).

Can we confirm \( \frac{d}{d\theta} \sin(\theta) = \cos(\theta) \)? Yes, we can.

\[
\frac{d}{d\theta} \sin(\theta) = \lim_{h \to 0} \frac{\sin(\theta+h)-\sin(\theta)}{h}.
\]

We want to simplify \( \sin(\theta h) \).

In general, can we say anything about \( \sin(\alpha+\beta) \)?

What about \( \cos(\alpha+\beta) \)?
\[
\sin(\alpha + \beta) = \sin(\alpha) \cos(\beta) + \cos(\alpha) \sin(\beta)
\]
\[
\cos(\alpha + \beta) = \cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta).
\]

Recall, we had:
\[
\frac{d}{d\theta} \sin(\theta) = \lim_{h \to 0} \frac{\sin(\theta + h) - \sin(\theta)}{h}
\]
\[
= \lim_{h \to 0} \frac{\sin(\theta) \cos(h) + \cos(\theta) \sin(h) - \sin(\theta)}{h}
\]
\[
= \lim_{h \to 0} \cos(\theta) \frac{\sin(h)}{h} + \lim_{h \to 0} \sin(\theta) \frac{\cos(h) - 1}{h}
\]
\[
= \cos(\theta).
\]
\[
\frac{\text{d}}{\text{d} \theta} \cos(\theta) = \lim_{h \to 0} \frac{\cos(\theta + h) - \cos(\theta)}{h} = \lim_{h \to 0} \frac{\cos(\theta)\cos(h) - \sin(\theta)\sin(h)}{h} = \lim_{h \to 0} \frac{\cos(h) - 1}{h} \cdot \frac{\sin(h)}{\sin(h)} = \lim_{h \to 0} \frac{\sin(h)}{h} = 1
\]
\[
\frac{d}{d\theta} \left( \tan^2(\theta) \right) = \frac{d}{d\theta} \left( \frac{\sin^2(\theta)}{\cos^2(\theta)} \right) = \sec^2(\theta).
\]