ASSIGNMENT 5

There are two parts to this assignment. The first part is on WeBWorK — the link is available on the course webpage. The second part consists of the questions on this page. You are expected to provide full solutions with complete justifications. You will be graded on the correctness and coherence of your solutions, as well as on their elegance. Your solutions must be typed, with your name and student number at the top of the first page. If your solutions are on multiple pages, the pages must be stapled together.

Your written assignment must be handed at the front of the lecture hall before the start of class on Monday, October 30. The two online assignments will close at 9:00 on Monday, October 30.

1. Imagine an hourglass consisting of two cones joined at the tip, as shown below. The cones are of height $H$ and radius $R$, and once the hourglass its tipped, sand flows from one cone to the other, where it settles flat, at a constant rate $C > 0$.

![Diagram of an hourglass](image)

Determine the rate of change of the height of sand in the bottom cone when half the sand has flowed out of the top cone. Your answer should be in terms of the constants $R$ and $C$.

2. In his 2008 textbook *Fundamentals of Multiphase Flows*, the author Christopher Brennen describes the expansion of a gas bubble in a superheated liquid. Under certain conditions — for example, in the absence of thermal effects — it may be determined that

$$\frac{dR}{dt} \approx \sqrt{\frac{2 (p_V - p_{\infty}^*)}{3 \rho_L}},$$

where $R$ is the radius of the bubble, $p_V$ is a constant describing vapour pressure within the bubble, $p_{\infty}^*$ is a constant describing pressure from the superheated liquid, and $\rho_L$ is a constant describing the density of the superheated liquid. Brennan writes that, under these conditions, the bubble experiences “explosive growth ... in which the volume displacement is increasing like $t^3$". Explain in three or four sentences what this means.