ASSIGNMENT 3

There are two parts to this assignment. The first part is on WeBWorK — the link is available on the course webpage. The second part consists of the questions on this page. You are expected to provide full solutions with complete justifications. You will be graded on the correctness and coherence of your solutions, as well as on their elegance. Your solutions must be typed, with your name and student number at the top of the first page. If your solutions are on multiple pages, the pages must be stapled together.

Your written assignment must be handed at the front of the lecture hall before the start of class on Monday, October 2. The online assignment will close at 9:00 on Monday, October 2.

1. The Lennard-Jones potential describes the potential energy $V(r)$ of a diatomic molecule consisting of two atoms a distance $r$ apart. It is given by

$$V(r) = \varepsilon \left( \left( \frac{R}{r} \right)^{12} - 2 \left( \frac{R}{r} \right)^6 \right),$$

where $\varepsilon$ and $R$ are positive constants. Because $r$ represents distance, the domain is restricted to $r > 0$.

   (a) Calculate $V'(r)$.

   (b) Explain, using your answer to part (a), why $V'(r) < 0$ for $0 < r < R$, $V'(R) = 0$ and $V'(r) > 0$ for $r > R$.

   (c) The three features of $V'(r)$ described in part (b) imply that the graph of $V(r)$ has a global minimum at the point $r = R$.\(^1\) Use this mathematical fact to explain in physical terms why $\varepsilon$ is referred to as the depth of the potential well.

2. Find two different functions whose graphs both have exactly three horizontal tangent lines, at $x = 0$, $x = 1$ and $x = 2$.

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\(^1\)This implication is intuitive but not obvious; it relies on a powerful result, which we do not prove here, called the Mean Value Theorem.