ASSIGNMENT 3  
Solutions

1. Evaluate the integral \( \int_0^2 9t^3 \sqrt{t^2 + 4} \, dt \).

   We make the substitution \( u = t^2 + 4 \), whence \( du = 2t \, dt \), \( t^2 = u - 4 \) and
   \[
   \int_0^2 9t^3 \sqrt{t^2 + 4} \, dt = \frac{9}{2} \int_4^8 (u - 4) \sqrt{u} \, du = \frac{9}{2} \left( \frac{2}{5} u^{5/2} - \frac{8}{3} u^{3/2} \right) \bigg|_4^8 = \frac{192}{5} (\sqrt{2} + 1).
   \]

2. Find all continuous functions \( f(t) \) such that
   \[
   \int_0^1 f(t) \, dt = \int_0^1 f(t^2)^2 \, dt + \frac{1}{3}.
   \]  (1)

   (Hint: write everything in terms of \( t^2 \).)

   For the integral on the left-hand side, we make the substitution \( u^2 = t \), whence \( 2u \, du = dt \) and
   \[
   \int_0^1 f(t) \, dt = \int_0^1 2uf(u^2) \, du = \int_0^1 2tf(t^2) \, dt.
   \]

   We may also write
   \[
   \frac{1}{3} = \int_0^1 t^2 \, dt.
   \]

   Thus we may rewrite (1) as
   \[
   \int_0^1 2tf(t^2) \, dt = \int_0^1 f(t^2)^2 \, dt + \int_0^1 t^2 \, dt,
   \]
   or
   \[
   0 = \int_0^1 \left( f(t^2)^2 - 2tf(t^2) + t^2 \right) \, dt = \int_0^1 \left( f(t^2) - t \right)^2 \, dt.
   \]

   It follows that \( f(t^2) = t \), or \( f(t) = \sqrt{t} \).

3. Periodically, you will be asked to post mathematical reflections on a blog. This term, the reflections are to be done in groups of 3-4. Your group must remain the same throughout all assignments, and the names of your group members should be written on each blog post.

   The task for your next post is as follows. It is a common error that students routinely conflate “integration” and “antidifferentiation”. Write a blog post explaining the difference between the two, and explaining why students commonly mistake one for the other. Your post should be 1 to 2 pages in length, and must include examples. It may also include illustrations and analogies. It is not sufficient to give definitions of the two terms: your post must address the connection between the two concepts.

   On your assignment submission, write down the URL of the blog where your post is located, along with the full names and student numbers of every member in your group.