ASSIGNMENT 6

There are two parts to this assignment. The first part is on WeBWorK — the link is available on the course webpage. The second part consists of the questions on this page. You are expected to provide full solutions with complete justifications. You will be graded on the mathematical, logical and grammatical coherence and elegance of your solutions. Your solutions must be typed, with your name and student number at the top of the first page. If your solutions are on multiple pages, the pages must be stapled together.

Your written assignment must be handed in before your recitation on Friday, October 27. The online assignment will close at 9:00 a.m. on Friday, October 27.

1. In recitations, you showed a limited version of the Power Rule; namely, that \( \frac{d}{dx} x^n = x^{n-1} \) for nonnegative integers \( n \). In this question, you extend the Power Rule to functions of the form \( x^{n/2} \) where \( n \) is a positive integer. You may not use any other version of the Power Rule in your proof.
   
   (a) Prove that \( \frac{d}{dx} x^{1/2} = \frac{1}{2} x^{-1/2} \).

   (b) Prove that if \( \frac{d}{dx} x^{k/2} = \frac{k}{2} x^{k/2-1} \) for a positive integer \( k \), then \( \frac{d}{dx} x^{(k+1)/2} = \frac{k+1}{2} x^{(k+1)/2-1} \).

   (c) Explain in a few sentences why your work in parts (a) and (b) imply that \( \frac{d}{dx} x^{n/2} = \frac{n}{2} x^{n/2-1} \) for all positive integers \( n \).

2. Choose the question on the midterm that was the hardest for you, and write a “model answer” for it. This should be an answer that is not only correct, but provides full explanations and a summary of the “key observation” you needed to make to solve the problem correctly.