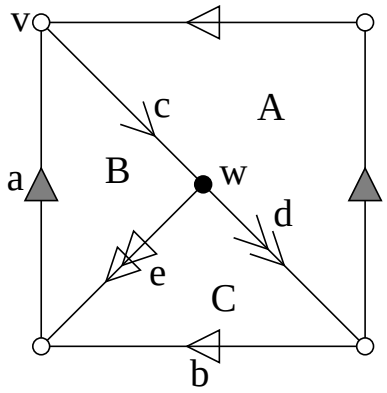
**Math 602 problems**

*Problem 0.1:* Let $M$ be the punctured plane $\mathbb{C}-\{0\}$. Determine the fundamental group $\pi_1$, the universal cover $F : U \to M$ and the deck transformations. For each subgroup of $G \subset \pi_1$ find a finite cover of $\mathbb{C} - \{0\}$ so that the group of covering transformations is $\pi_1/G$.

*Problem 0.2:* Show that the twice punctured plane $\mathbb{C} - \{p, q\}$ cannot be parabolic, that is, covered by $\mathbb{C}$. (Hint: what do you know about the range of an entire function?)

*Problem 0.3:* Consider the following "triangulation" of the torus. (It is not quite a valid triangulation, because the triangles don't just intersect along a common edge.)




A 2-chain $n_1A + n_2B + n_3C$ can be represented by the vector $\begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$. Similarly, 1-chains and 0-chains can be represented by vectors with 5 and 2 integer components respectively. With respect to these representations, the boundary operators $\delta$ (usually denoted $\partial$) act as matrices. Find the matrices, and verify that $\delta \circ \delta = 0$. Give an explicit description of the closed and exact chains, and compute the homology groups.

*Problem 0.4:* Consider the torus $T = \mathbb{C}/\Gamma$, where $\Gamma$is a lattice with periods 1 and $\tau$. Show that the space of holomorphic 1-forms on $T$ is one dimensional. (Hint: consider the lift to the covering space.) If such a holomorphic one form $\omega$ is normalized so that the integral along the cycle running from 0 to 1 is equal to 1, what is the integral of $\omega$ along the cycle running from 0 to $\tau$?