Integration Formulae

In the following
  ○ $f, g$ are $C^1$ functions.
  ○ $\varphi, \psi$ are $C^2$ functions.
  ○ $\omega$ is a $C^1$ 1–form.
  ○ $D$ is a region in $M$ with compact closure and piecewise differentiable boundary.

(1) $d\omega = 0 \Rightarrow \int_{\delta D} \omega = 0$

(2) $\int_{\delta D} f\omega = \int_{D} df \wedge \omega + \int_{D} f d\omega$

(3) $f\omega$ has compact support contained in $D \Rightarrow \int_{D} df \wedge \omega + \int_{D} f d\omega = 0$

(4) $(df, *\omega)_D = \int_{D} fd\bar{\omega} - \int_{\delta D} f\bar{\omega}$

(5) $(d\varphi, d\psi)_D = -\int_{D} \varphi \Delta \bar{\psi} + \int_{\delta D} \varphi * d\bar{\psi}$

(6) $\int_{D} (\varphi \Delta \psi - \psi \Delta \varphi) = \int_{\delta D} (\varphi * d\psi - \psi * d\varphi)$

(7) $(df, *d\psi)_D = -\int_{\delta D} f d\bar{\psi}$

(8) $d\omega = 0 \Rightarrow \int_{\delta D} f\bar{\omega} = \int_{D} df \wedge \bar{\omega}$

(9) $\int_{\delta D} f d\bar{g} = -\int_{\delta D} \bar{g} df$

(10) $d\omega = d(f\omega) = 0 \Rightarrow \int_{D} df \wedge \bar{\omega} = 2 \int_{\delta D} (\text{Re } f) \bar{\omega} = 2i \int_{\delta D} (\text{Im } f) \bar{\omega}$