

Urysohn's Lemma

We constructed open sets V_r , $r \in \mathbb{Q} \cap [0, 1]$, obeying

$$K \subset V_1 \subset \overline{V_1} \subset \cdots \subset V_s \subset \overline{V_s} \subset \cdots \subset V_r \subset \overline{V_r} \subset \cdots \subset V_0 \subset \overline{V_0} \subset V \subset X$$

for all $0 < r < s < 1$, $r, s \in \mathbb{Q} \cap [0, 1]$. Then we defined

$$f(x) = \begin{cases} \sup \{ r \mid r \in \mathbb{Q} \cap [0, 1], x \in V_r \} & \text{if } x \in V_0 \\ 0 & \text{if } x \notin V_0 \end{cases}$$

