

# Vector Identities

## Gradient

1.  $\vec{\nabla}(f + g) = \vec{\nabla}f + \vec{\nabla}g$
2.  $\vec{\nabla}(cf) = c\vec{\nabla}f$ , for any **constant**  $c$
3.  $\vec{\nabla}(fg) = f\vec{\nabla}g + g\vec{\nabla}f$
4.  $\vec{\nabla}(f/g) = (g\vec{\nabla}f - f\vec{\nabla}g)/g^2$  at points  $\vec{x}$  where  $g(\vec{x}) \neq 0$ .
5.  $\vec{\nabla}(\vec{F} \cdot \vec{G}) = \vec{F} \times (\vec{\nabla} \times \vec{G}) - (\vec{\nabla} \times \vec{F}) \times \vec{G} + (\vec{G} \cdot \vec{\nabla})\vec{F} + (\vec{F} \cdot \vec{\nabla})\vec{G}$

## Divergence

6.  $\vec{\nabla} \cdot (\vec{F} + \vec{G}) = \vec{\nabla} \cdot \vec{F} + \vec{\nabla} \cdot \vec{G}$
7.  $\vec{\nabla} \cdot (c\vec{F}) = c\vec{\nabla} \cdot \vec{F}$ , for any **constant**  $c$
8.  $\vec{\nabla} \cdot (f\vec{F}) = f\vec{\nabla} \cdot \vec{F} + \vec{F} \cdot \vec{\nabla}f$
9.  $\vec{\nabla} \cdot (\vec{F} \times \vec{G}) = \vec{G} \cdot (\vec{\nabla} \times \vec{F}) - \vec{F} \cdot (\vec{\nabla} \times \vec{G})$

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10.  $\vec{\nabla} \times (\vec{F} + \vec{G}) = \vec{\nabla} \times \vec{F} + \vec{\nabla} \times \vec{G}$
11.  $\vec{\nabla} \times (c\vec{F}) = c\vec{\nabla} \times \vec{F}$ , for any **constant**  $c$
12.  $\vec{\nabla} \times (f\vec{F}) = f\vec{\nabla} \times \vec{F} + \vec{\nabla}f \times \vec{F}$
13.  $\vec{\nabla} \times (\vec{F} \times \vec{G}) = \vec{F}(\vec{\nabla} \cdot \vec{G}) - (\vec{\nabla} \cdot \vec{F})\vec{G} + (\vec{G} \cdot \vec{\nabla})\vec{F} - (\vec{F} \cdot \vec{\nabla})\vec{G}$

## Laplacian

14.  $\vec{\nabla}^2(f + g) = \vec{\nabla}^2f + \vec{\nabla}^2g$
15.  $\vec{\nabla}^2(cf) = c\vec{\nabla}^2f$ , for any **constant**  $c$
16.  $\vec{\nabla}^2(fg) = f\vec{\nabla}^2g + 2\vec{\nabla}f \cdot \vec{\nabla}g + g\vec{\nabla}^2f$

## Degree Two

17.  $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{F}) = 0$
18.  $\vec{\nabla} \times (\vec{\nabla}f) = 0$
19.  $\vec{\nabla} \cdot (\vec{\nabla}f \times \nabla g) = 0$
20.  $\vec{\nabla} \cdot (f\vec{\nabla}g - g\vec{\nabla}f) = f\vec{\nabla}^2g - g\vec{\nabla}^2f$
21.  $\vec{\nabla} \times (\vec{\nabla} \times \vec{F}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{F}) - \vec{\nabla}^2\vec{F}$