MATH 317 PROBLEM SET IX

Due Friday, April 4

1) Let C be the curve of intersection of the cylinder $x^2 + y^2 = 1$ and the surface $z = y^2$ oriented in the counterclockwise direction as seen from $(0, 0, 100)$. Let $\vec{F} = (x^2 - y, y^2 + x, 1)$. Calculate $\oint_C \vec{F} \cdot d\vec{r}$
   a) by direct evaluation
   b) by using Stokes' Theorem.
2) Let S be the unit sphere centered at the origin and oriented by the outward pointing normal. If

$$\vec{F}(x, y, z) = \big(x, y, z^2\big)$$

   evaluate the flux of $\vec{F}$ through S
   a) directly and
   b) by applying the Divergence Theorem.
3) Let S be the oriented surface consisting of the top and four sides of the cube whose vertices are $(\pm 1, \pm 1, \pm 1)$. oriented outward. If $\vec{F}(x, y, z) = (xyz, xy^2, x^2yz)$, find the flux of $\vec{\nabla} \times \vec{F}$ through S.
4) Let S denote the part of the spiral ramp (that is helicoidal surface) parametrized by $x = u \cos v,\ y = u \sin v,\ z = v$ with $0 \le u \le 1$ and $0 \le v \le 2\pi$. Let C denote the boundary of S with orientation specified by the upward pointing normal on S. Find

$$\int_C y\, dx - x\, dy + xy\, dz$$

5) Let S be a closed surface whose top is the disc $x^2 + y^2 \le 1$ in the xy–plane and whose bottom is a piecewise smooth bag hanging below the xy–plane from the unit circle. If the volume of the bag is 12 cc, what is the flux through the bag of the vector field $\vec{F} = (3yz^4 + 4x)\hat{\imath} + (6xz - y)\hat{\jmath} + y^2\hat{\mathbf{k}}$?
6) Let $f : [0, 1] \to [0, \infty)$ be a continuously differentiable strictly decreasing function such that $f(1) = 0$. Let S_1 be the surface $\big\{ (x, y, z) \bigm| z = f(x^2 + y^2),\ x^2 + y^2 \le 1 \big\}$ and let S_2 be the disk $\big\{ (x, y, 0) \bigm| x^2 + y^2 \le 1 \big\}$. Let $S = S_1 \cup S_2$. Orient S by the "outward normal". The flux of $\vec{F}(x, y, z) = (x, xy + 2y, yz)$ out of S is measured to be 12. Find the volume enclosed by S.
7) Let C be any piecewise smooth path from $(1, 1, 1)$ to $(3, 2, 1)$. Find all real numbers α, β, γ such that

$$\int_C (\alpha y + \beta z)\, dx + (\gamma x + 2z)\, dy + (\alpha x - \beta y)\, dz$$

   is independent of C and evaluate the integral.

Reminder: The final exam is on Tuesday, April 15 at 3:30pm in HEBB TH.