

MATH 317 PROBLEM SET VIII

Due Friday, March 28

- 1) Evaluate, both by direct integration and by Stokes' Theorem, $\oint_C (z dx + x dy + y dz)$ where C is the circle $x + y + z = 0$, $x^2 + y^2 + z^2 = 1$. Orient C so that its projection on the xy -plane is counterclockwise.
- 2) Evaluate $\oint_C (x \sin y^2 - y^2) dx + (x^2 y \cos y^2 + 3x) dy$ where C is the counterclockwise boundary of the trapezoid with vertices $(0, -2)$, $(1, -1)$, $(1, 1)$ and $(0, 2)$.
- 3) Evaluate $\oint_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = ye^x \hat{i} + (x + e^x) \hat{j} + z^2 \hat{k}$ and C is the curve

$$\vec{r}(t) = (1 + \cos t) \hat{i} + (1 + \sin t) \hat{j} + (1 - \sin t - \cos t) \hat{k}$$

- 4) Let C be the intersection of $x + 2y - z = 7$ and $x^2 - 2x + 4y^2 = 15$. The curve C is oriented counterclockwise when viewed from high on the z -axis. Let

$$\vec{F} = (e^{x^2} + yz) \hat{i} + (\cos(y^2) - x^2) \hat{j} + (\sin(z^2) + xy) \hat{k}$$

Evaluate $\oint_C \vec{F} \cdot d\vec{r}$.

- 5) Consider $\iint_S (\vec{\nabla} \times \vec{F}) \cdot \hat{n} dS$ where S is the portion of the sphere $x^2 + y^2 + z^2 = 1$ that obeys $x + y + z \geq 1$, \hat{n} is the upward pointing normal to the sphere and $\vec{F} = (y - z) \hat{i} + (z - x) \hat{j} + (x - y) \hat{k}$. Find another surface S' with the property that $\iint_S (\vec{\nabla} \times \vec{F}) \cdot \hat{n} dS = \iint_{S'} (\vec{\nabla} \times \vec{F}) \cdot \hat{n} dS$ and evaluate $\iint_{S'} (\vec{\nabla} \times \vec{F}) \cdot \hat{n} dS$.
- 6) Verify the identity $\oint_C \phi \vec{\nabla} \psi \cdot d\vec{r} = - \oint_C \psi \vec{\nabla} \phi \cdot d\vec{r}$ for any continuously differentiable scalar fields ϕ and ψ and curve C that is the boundary of a piecewise smooth surface.
- 7) Use Green's Theorem to show that

$$\iint_R \vec{\nabla} \cdot \vec{F} dA = \oint_C \vec{F} \cdot \hat{n} ds$$

where C is the oriented boundary of the plane domain R , $\vec{F} = F_1 \hat{i} + F_2 \hat{j}$, \hat{n} is the outward normal to C and s is the arclength along C .

Reminder: The final exam is on Tuesday, April 15 at 3:30pm.