1. Evaluate, by two methods, the integral \( \iint_S \mathbf{F} \cdot \hat{n} \, dS \), where \( \mathbf{F} = z \hat{k} \), \( S \) is the surface \( x^2 + y^2 + z^2 = a^2 \) and \( \hat{n} \) is the outward pointing unit normal to \( S \).
   (a) First, by direct computation of the surface integral.
   (b) Second, by using the divergence theorem.

2. Let \( \mathbf{F} = z y^3 \hat{i} + y x \hat{j} + (2z + y^2) \hat{k} \).
   Let \( V \) be the solid in 3-space defined by
   \[
   0 \leq z \leq \frac{9 - x^2 - y^2}{9 + x^2 + y^2}
   \]
   Let \( D \) be the bottom surface of \( V \). It is \( z = 0, \ x^2 + y^2 \leq 9 \).
   Let \( S \) be the curved portion of the boundary of \( V \). It is \( z = \frac{9 - x^2 - y^2}{9 + x^2 + y^2}, \ x^2 + y^2 \leq 9 \).

   Denote by \( |V| \) the volume of \( V \) and compute, in terms of \( |V| \),
   (a) \( \iint_D \mathbf{F} \cdot \hat{n} \, dS \) with \( \hat{n} \) pointing downward
   (b) \( \iiint_V \nabla \cdot F \, dV \)
   (c) \( \iint_S \mathbf{F} \cdot \hat{n} \, dS \) with \( \hat{n} \) pointing outward
   Use the divergence theorem to answer at least one of parts (a), (b) and (c).

3. Evaluate the integral \( \iint_S \mathbf{F} \cdot \hat{n} \, dS \), where \( \mathbf{F} = (x, y, 1) \) and \( S \) is the surface \( z = 1 - x^2 - y^2 \), for \( x^2 + y^2 \leq 1 \), by two methods.
   (a) First, by direct computation of the surface integral.
   (b) Second, by using the divergence theorem.

4. (a) By applying the divergence theorem to \( \mathbf{F} = \phi \mathbf{a} \), where \( \mathbf{a} \) is an arbitrary constant vector, show that
   \[
   \iiint_V \nabla \phi \, dV = \iint_{\partial V} \phi \hat{n} \, dS
   \]
(b) Show that the centroid \((\bar{x}, \bar{y}, \bar{z})\) of a solid \(V\) with volume \(|V|\) is given by

\[
(\bar{x}, \bar{y}, \bar{z}) = \frac{1}{2|V|} \int \int \int_{\partial V} (x^2 + y^2 + z^2) \hat{n} dS
\]

5. Find the flux of \(F = (y + xz)\hat{i} + (y + yz)\hat{j} - (2x + z^2)\hat{k}\) upward through the first octant part of the sphere \(x^2 + y^2 + z^2 = a^2\).

6. (Optional — not to be handed in) Let \(F = (x - yz)\hat{i} + (y + xz)\hat{j} + (z + 2xy)\hat{k}\) and let

- \(S_1\) be the portion of the cylinder \(x^2 + y^2 = 2\) that lies inside the sphere \(x^2 + y^2 + z^2 = 4\)
- \(S_2\) be the portion of \(x^2 + y^2 + z^2 = 4\) that lies outside the cylinder \(x^2 + y^2 = 2\)
- \(V\) be the solid bounded by \(S_1\) and \(S_2\)

Compute

(a) \(\int \int_{S_1} F \cdot \hat{n} dS\) with \(\hat{n}\) pointing inward
(b) \(\int \int \int_V \nabla \cdot F dV\)
(c) \(\int \int_{S_2} F \cdot \hat{n} dS\) with \(\hat{n}\) pointing outward

Use the divergence theorem to answer at least one of parts (a), (b) and (c).

7. (Optional — not to be handed in) Let \(E(r)\) be the electric field due to a charge configuration that has density \(\rho(r)\). Gauss’ law states that, if \(V\) is any solid in \(\mathbb{R}^3\) with surface \(\partial V\), then the electric flux

\[
\int \int_{\partial V} E \cdot \hat{n} dS = 4\pi Q \quad \text{where} \quad Q = \int \int \int_V \rho dV
\]

is the total charge in \(V\). Here, as usual, \(\hat{n}\) is the outward pointing unit normal to \(\partial V\). Show that

\[
\nabla \cdot E(r) = 4\pi \rho(r)
\]

for all \(r\) in \(\mathbb{R}^3\). This is one of Maxwell’s equations. Assume that \(\nabla \cdot E(r)\) and \(\rho(r)\) are well-defined and continuous everywhere.

Reminder: Midterm II is scheduled for Wednesday, March 14.

Reminder: The final exam is on Tuesday, April 24 at 7:00pm.